NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination 2017-18

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

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Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Point A lies at $(-5,\ 6)$ and point B lies at $(7,\ 2)$.

Find the equation of the line which is perpendicular to AB and passes through point B.

Give your answer in the form $ax + by + c = 0$ where $a,\ b$ and c are integers. **[4]**

Question A2

The probability that a machine develops a fault on Wednesday is $\frac{2}{5}$. If it develops a fault on Wednesday, the probability that it develops a fault on Thursday is $\frac{3}{10}$. If the machine does not develop a fault on Wednesday, the probability that it develops a fault on Thursday is $\frac{5}{8}$.

a) Draw a fully labelled tree diagram. **[2]**

b) For a given Wednesday and Thursday, find the probability that the machine develops a fault on exactly one of these days **[3]**

Question A3

Find the range of values which satisfy $3x^2 - 4x - 15 > 0$. **[4]**

Question A4

A geometric series is defined as $8 + 12 + 18 + \cdots$

How many terms of the series are needed for the sum to exceed 15000? **[4]**

Question A5

Solve the equation

$$\log_2(x^2 + 4x) - \log_2(x^2 - 16) = 2 \qquad (x > 4)$$ **[4]**

All working must be shown.

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Question A6

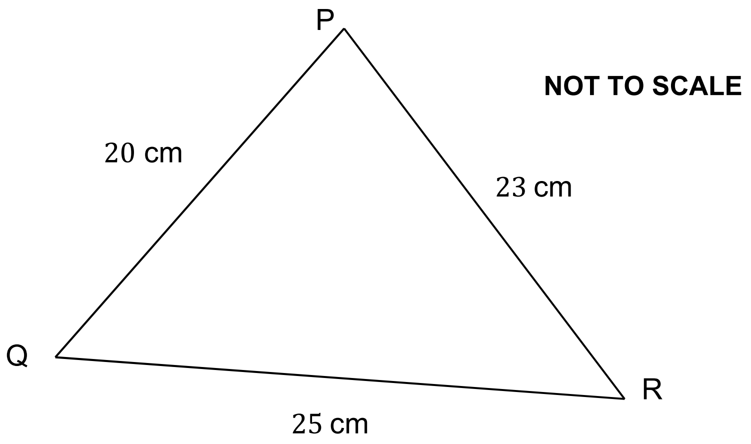

Figure 1

Figure 1 shows triangle PQR with PQ = 20 cm, PR = 23 cm and QR = 25 cm. Find the size of angle Q, giving your answer in **radians** and to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[4]**

Question A7

Find the equation of the tangent to the curve $y = 3x^2 - 10x + 7$ at the point $(2, -1)$. **[3]**

Question A8

Function $f(x)$ is defined as $f(x) = e^x + 1 \quad (-\infty < x < \infty)$

Function $g(x)$ is defined as $g(x) = x^2 \quad (-\infty < x < \infty)$

Find $g(f(\ln 3))$. **[3]**

Question A9

Prove that $\sin 2A(\cot A - \tan A) = 2\cos 2A$.

__Each__ stage of your working must be clearly shown. **[3]**

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Question A10

Use integration by parts to evaluate

$$\int_0^{\frac{\pi}{8}} 8x \cos 4x \ \ dx.$$

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[5]**

Question A11

The masses of bags of rock salt can be assumed to follow a Normal distribution with standard deviation 2 kg.

A sample of 10 bags is selected and the mean mass was found to be 24.9 kg. Find a 95% confidence interval of the mean mass of all of the bags. **[3]**

Question A12

The table below shows the values of $\frac{1}{1\ +\ e^x}$ (given to 3 decimal places) for $x = 1,\ \ 1.25,\ \ 1.5,\ \ 1.75,$ and 2.

$\boldsymbol{x}$	1	1.25	1.5	1.75	2
$\boldsymbol{\frac{1}{1\ +\ e^x}}$	0.269	0.223	0.182	0.148	0.119

Use the trapezium rule with 4 intervals to find an approximate value of

$$\int_1^2 \frac{1}{1+\ e^x}\ \ dx.$$

[3]

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Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) Use substitution to solve the equations $x + 2y = 5$

$$x^2 + 4y^2 = 73$$ **[6]**

b) i. Use the Factor theorem to show that $(x + 4)$ is a factor of $6x^3 + 17x^2 - 31x - 12$. **[2]**

ii. Divide $6x^3 + 17x^2 - 31x - 12$ by $(x + 4)$. **[3]**

c) A supplier has 1100 bolts in stock at the beginning of June. On 1 June, he sells 8 bolts and on each subsequent day sells 2 more bolts than the previous day *i.e.* he sells 8 bolts on 1 June, 10 bolts on 2 June, 12 bolts on 3 June, and so on.

i. On which day does he sell 44 bolts? **[2]**

ii. Does he have enough bolts in stock to last the month (30 days)? **[3]**

Show your working.

d) In the expansion of $(2 + px)^8$, where $p \neq 0$, the coefficient of the term in x^2 is 4 times larger than the coefficient of the term in x^3.

Find the value of p. **[4]**

Question B2 is on the next page.

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Question B2

a) Two variables, p and t, are connected by the formula

$$p = A\left(4096^{kt}\right) + 48$$

When $t = 0$, $p = 48\frac{1}{2}$.

i. Show that $A = \frac{1}{2}$. **[1]**

When $t = 6$, $p = 80$.

ii. Find the value of k, giving your answer in the form $\frac{1}{n}$ where n is an integer. *All working must be shown.* **[3]**

iii. Find the value of p when $t = 4$. **[2]**

b) Solve $(\ln x)^2 + 3\ln x = 10$. **[4]**

c) Solve the equation $\tan\left[2\left(x - \frac{\pi}{3}\right)\right] = \frac{1}{\sqrt{3}}$ $\quad (0 \le x \le 2\pi)$

Give your answers as exact multiples of π. **[5]**

Question B2 continues on the next page.

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d)

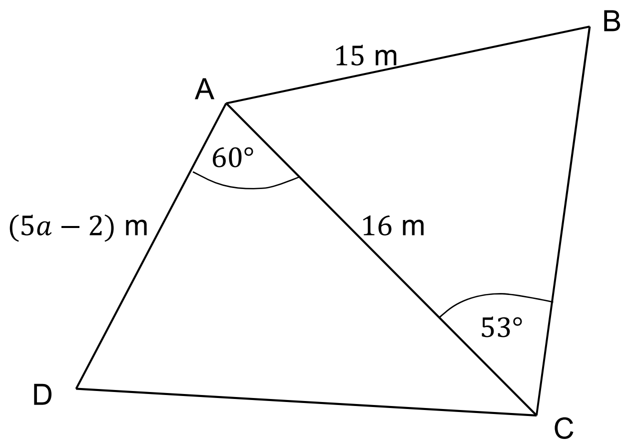

Figure 2

Figure 2 shows the quadrilateral ABCD which is made up of two acute-angled triangles ABC and ACD. AD $= (5a - 2)$ m, AC $= 16$ m and AB $= 15$ m. Angle ACB $= 53°$ and angle DAC $= 60°$.
The area of triangle ACD $= 72\sqrt{3}$ m^2.

i. Find the value of a. **[3]**

ii. Find the size of angle B. **[2]**

Question B3 is on the next page.

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Question B3

a) The equation of a curve is given by $y = 3x^4 - 8x^3 + 9$.

i. Find $\frac{dy}{dx}$. **[2]**

ii. Show that there are stationary values at $x = 0$ and $x = 2$. **[2]**

iii. Confirm that there is a point of inflexion at $x = 0$ and determine whether the stationary value at $x = 2$ is a maximum or a minimum. **[4]**

iv. Sketch the curve $y = 3x^4 - 8x^3 + 9$. **(This must not be done on graph paper.)** On your sketch, show clearly the coordinates where the curve crosses the $y -$ axis; and the coordinates of the stationary values. (You do **not** have to show where the curve crosses the $x -$ axis.) **[3]**

b) i. Find

$$\int (3 - \frac{2}{x})^2 \, dx.$$

[4]

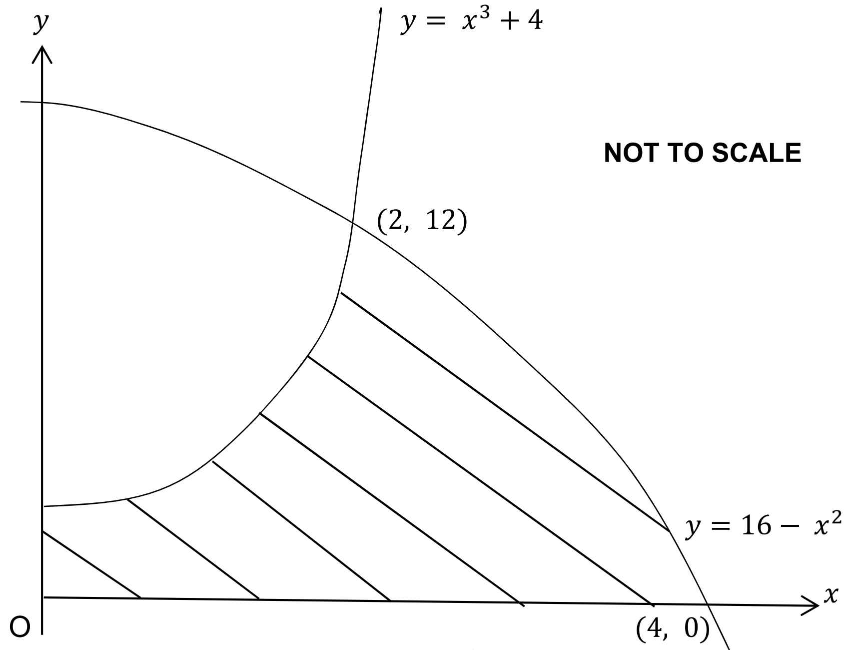

Figure 3

Figure 3 shows the curves $y = 16 - x^2$ and $y = x^3 + 4$.

The curve $y = 16 - x^2$ meets the $x -$ axis at $(4, 0)$ and intersects with the curve $y = x^3 + 4$ at the point $(2, 12)$.

ii. Find the area, which is shaded on the diagram, that is bounded by both curves and the $x -$ and $y -$ axes. *All working must be shown*. **[5]**

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Question B4

a) The function $f(x)$ is defined as $f(x) = e^{2x} + 1 \quad (-\infty < x < \infty)$.

i. State the range of $f(x)$. **[1]**

ii. Write down an expression for $f^{-1}(x)$. **[2]**

iii. State the domain of $f^{-1}(x)$. **[1]**

The function $g(x)$ is defined as $g(x) = \frac{5x - 1}{3} \quad (-\infty < x < \infty)$.

iv. Solve $g(x) = g^{-1}(x)$. **[3]**

b)

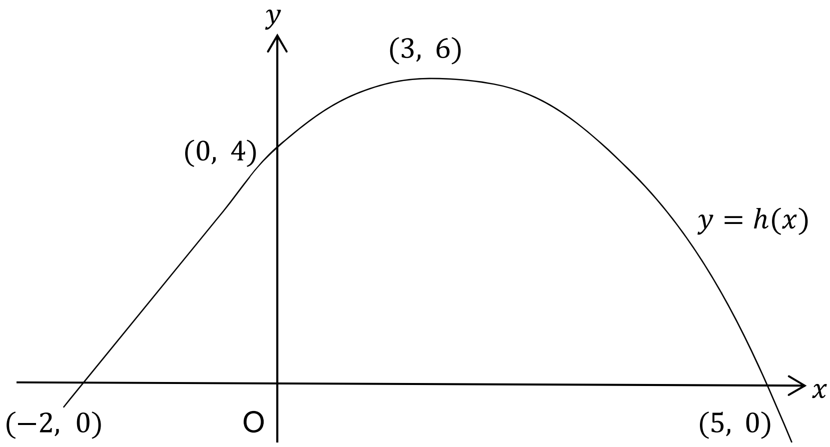

Figure 4

Figure 4 shows the graph of $y = h(x)$ which crosses the $y -$ axis at $(0,\ 4)$ and the $x -$ axis at $(-2,\ 0)$ and $(5,\ 0)$. There is a stationary value at $(3,\ 6)$.

On two separate sets of axes, draw sketches of the following. On each sketch show clearly the coordinates of any stationary values, and where the curve crosses the $x -$ axis and the $y -$ axis.

i. $y = \frac{1}{2}h(x)$. **[3]**

ii. $y = h(x - 2)$. **[3]**

c) i. Use one of the addition formulae to prove that $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$. **[2]**

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ii. Solve the equation $\tan 2\theta + \tan\theta = 0 \quad (0° \leq \theta \leq 180°)$ **[5]**

Question B5

a) Line l has vector equation $\boldsymbol{r} = (5\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}) + \mu(-3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$ where μ is a scalar.

i. Show that point $A(2,\ 2,\ 3)$ lies on line l. **[2]**

You are given point $B(14, -14,\ 7)$ lies on line l and that point $C(10, -16,\ 11)$ does <u>not</u> lie on line l.

ii. Find the acute angle between CA and CB. **[3]**

iii. Find the area of triangle ACB. **[2]**

iv. By using the scalar product, show that CB is perpendicular to line l. **[1]**

v. If the vector $\boldsymbol{a} = \overrightarrow{AB}$, find the unit vector $\boldsymbol{a}$. **[2]**

vi. Explain why $AB^2 + BC^2 = AC^2$. **[2]**

b) A curve has equation $4x^2 - 2xy + 3y = 9$.

i. Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

ii. Find the equation of the normal to the curve at the point $(2,\ 7)$. **[3]**

iii. A relationship between x and y is given by $y = kx$ when there is a stationary value on the curve.

State the value of k. **[1]**

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Question B6

a) A curve has equation

$$y = \frac{x^2 + 1}{x + 1}$$

Use the Quotient Rule to find $\frac{dy}{dx}$ and hence find the coordinates of the points where the gradient of the curve is $\frac{7}{8}$. **[5]**

b) i. Express $\frac{1}{(x+2)(x+3)}$ in the form $\frac{A}{x + 2} + \frac{B}{x + 3}$ where A and B are constants to be determined. **[3]**

ii. Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{(x + 2)(x + 3)}$$

subject to $y = \frac{4}{3}$ when $x = 0$. **[5]**

Write your answer in the form $y = f(x)$ which must contain no logarithms.

c) i. Use the substitution $u = x^2 + 4x - 3$ to find **[4]**

$$\int (x + 2)(x^2 + 4x - 3)^4 \, dx.$$

The curve $y = (x + 2)^{½}(x^2 + 4x - 3)^2$ is rotated about the $x -$ axis between $x = 0$ and $x = 1$.

ii. Find the volume formed.

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[3]**

This is the end of the examination.

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