THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A5. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

a) $c = 8$ [can be seen anywhere in the solution] **[B1]**

Attempts to find the gradient of the line **[M1]**

$m = \frac{1}{2}$ or equivalent **[A1]**

b) $p = -16$ [allow follow through] **[A1ft]**

Question A2

$\frac{7}{11}, \frac{6}{10}, \frac{5}{9}$ any one correct fraction or equivalent decimal seen **[M1]**

Multiplies their probabilities **[M1]**

$= \frac{7}{33}$ or equivalent, or anything rounding to 0.212 **[A1]**

Question A3

Please note: the Remainder Theorem must be used: any other method scores 0.

Substitutes $x = \frac{1}{2}$ into the expression **[M1*]**

Sets equal to 1 and finds a value for k. **[M1]**

$k = 1$ **[A1]**

Special case: if a candidate finds $f(1)$, sets this equal to 1 and then finds a value for k, award 1 mark out of 3.

Question A4

$ar = 3000$ and $ar^6 = 7.29$ **[M1]**

Divides (either way round) and reaches $r^5 \text{ or } r^{-5} = \cdots \left(\frac{243}{100000} \text{ or } \frac{100000}{243}\right)$ **[M1]**

$r = \frac{3}{10}$ or equivalent **[A1]**

Substitutes their r into either expression **[M1]**

$a = 10000$ **[A1]**

Question A5

Takes logs correctly $(x \log 8 = \log 2700)$ **[M1]**

$x = 3.79958\ldots$ (can be implied) **[A1]**

$= 3.80$ to 3 significant figures. (Allow follow through) **[A1ft]**

Question A6

$2\theta = 32,\ 148,\ 392,\ 508$ Any one correct. Allow answers outside the range. **[M1]**

Realises the need to search from $0°$ to $720°$ for 2θ **[M1]**

Divides by 2 at the right time **[M1]**

$\theta = 16,\ 74,\ 196,\ 254$ (degrees) **(A1)** for any two correct; **(A2)** all correct. **[A2]**
[Any extra solutions in the range results in the loss of 1 mark; ignore solutions outside the range]

Question A7

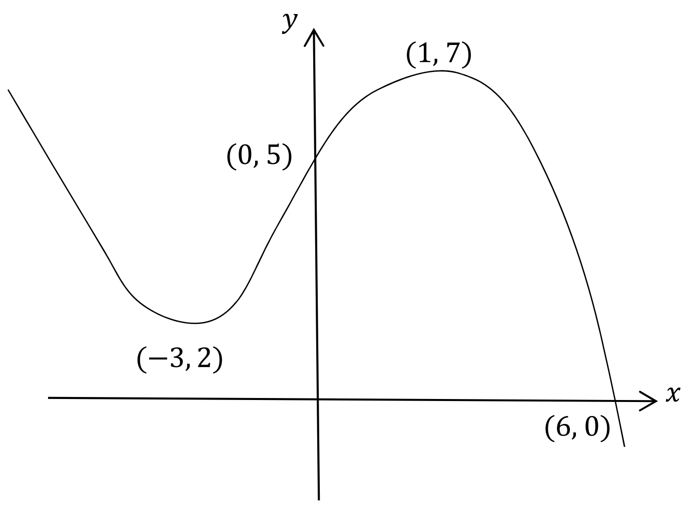
Differentiates (sight of e^{2x} is sufficient for this mark) $(= 2e^{2x})$ **[M1*]**

Substitutes $x = \ln 3$ into their $\frac{dy}{dx}$ $(= 18)$ **[M1]**

Substitutes $x = \ln 3$ into original equation to find a value for y $(= 15)$ **[M1]**

$y - 15 = 18(x - \ln 3)$ or equivalent **[A1]**

Question A8



Correct shape **[B1]**

(0,5) and (6,0) shown **[B1]**

(1,7) and (-3,2) shown **[B1]**

Question A9

$z = \frac{300 - 290}{5}$ $(= 2)$ **[M1]**

Finds ϕ(their 2) $(=0.9772)$ and subtracts from 1 (0.0228) **[M1]**

Anything rounding to 2.3% (must be a percentage) **[A1]**

Question A10

$8x + 3x^2y + x^3\frac{dy}{dx} - 2y^3\frac{dy}{dx} = 0$

Correct use of Product Rule (sight of $\pm 3x^2y \pm x^3\frac{dy}{dx}$ is sufficient for this mark) **[M1*]**

Correct implicit differentiation (sight of $\pm x^3\frac{dy}{dx}$ or $\pm 2y^3\frac{dy}{dx}$ is sufficient) **[M1*]**

Gathers $\frac{dy}{dx}$ terms on to one side and factorises (only available if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{-8x - 3x^2y}{x^3 - 2y^3}$ or equivalent **[A1]**

Question A11

Takes scalar product **[M1]**

Sets equal to 0 and finds a value for p **[M1]**

$p = 3$ **[A1]**

Question A12

Uses integration by parts in the right direction **[M1*]**

$128x^2 \times \frac{1}{4}e^{4x}$ **(A1)** $- \int \underline{256x \times \frac{1}{4}e^{4x}\ dx}$ (A1) for first part only **[A1]**

Uses integration by parts again **[M1*]**

$32x^2e^{4x} - [\, \underline{256x \times \frac{1}{16}e^{4x}}$ **(A1ft)** $- \int 256 \times \frac{1}{16}e^{4x}\ dx\ ]$ **[A1ft]**

The (A1ft) is for a correct first part from integrating the expression in the second line (relevant sections underlined)

$32x^2e^{4x} - 16xe^{4x} + 4e^{4x} + c$ **[A1]**

Section B

Question B1

a) Solves to find two critical values $(0, \frac{1}{2})$ **[M1]**

$x \geq 0$ **(A1)*** $x \leq \frac{1}{2}$ **(A1)*** or $0 \leq x \leq \frac{1}{2}$ **(A1)** for each end **[A2]**

<u>*Please note</u>: if this version of the answers is quoted, the two ranges can be separated by a space, a comma or the word 'and'. The final mark is lost if the word 'or' is seen.

b) i.

$$
\begin{array}{r l}
 & x^2 - x - 6 \\
x - 2 \,\big) & x^3 - 3x^2 - 4x + 12 \\
 & x^3 - 2x^2 \\
 & \quad -x^2 - 4x \\
 & \quad -x^2 + 2x \\
 & \qquad -6x + 12 \\
 & \qquad -6x + 12 \\
 & \qquad \cdots\cdots\cdots
\end{array}
$$

correct first division **[M1]**

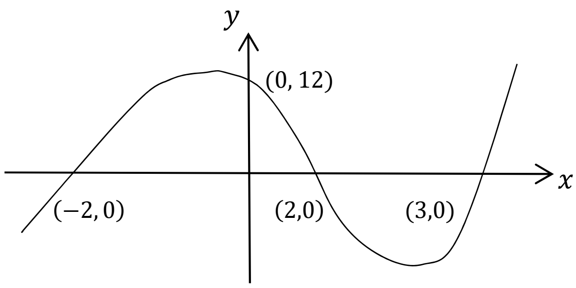
any correct subsequent division **[M1]**

correct quotient **[A1]**

ii. $(x + 2)(x - 2)(x - 3)$ **[A1]**

iii. $x = -2,\ 2,\ 3$ **[A1ft]**

iv.



Correct shape, do not worry about position of maximum **[B1]**

(-2,0); (2,0); (3,0) **[B1ft]**

(0,12) **[B1]**

c) i. $a + 9d = 3 \times (a + 2d)$ **[M1*]**

At least one line of intermediate working followed by $a = \frac{3}{2}d$ **[A1]**

ii. $5040 = \frac{40}{2}\left[2 \times \frac{3}{2}d + 39 \times d\right]$ or $\frac{40}{2}[2a + 39 \times \frac{2}{3}a]$ **[M1]**

calculates correctly in right order **[M1]**

Finds a value for d or a **[M1]**

Substitutes in any expression **[M1]**

$a = 9, \quad d = 6$ **[A1]**

d) ${}^{7}C_4 \times 3^3 \times (-2)^4$ (Allow ${}^{4}C_7$ and presence of x) **[M1]**

15120 **[A1]**

Question B2

a) i. 80000 **[B1]**

ii. Substitutes $t = 5$ into expression **[M1]**

Anything rounding to 86200 **[A1]**

iii. $\frac{dN}{dt} = 80000 \times 0.015 \times e^{0.015t}$ (No need to simplify) **[B1]**

iv. Substitutes into their $\frac{dN}{dt}$ and reaches $e^{0.015t} = \cdots (\frac{1313}{1200})$ **[M1]**

Takes logs and reaches $0.015t = \cdots [\ln\left(\frac{1313}{1200}\right)]$ **[M1]**

$t =$ anything rounding to 6 years **[A1]**

b) $\log_3\left[\frac{x(x+5)}{(x+5)(x-5)}\right] = \log_3 3$

Uses the subtraction law for logs **[M1*]**

Adapts the 1 and removes logs at the right time **[M1*]**

Solves $[\frac{x}{x-5} = 3]$ **[M1]**

$x = \frac{15}{2}$ or equivalent **[A1]**

[If the cancelling is missed, the third M mark can be scored for forming a quadratic equation $(2x^2 - 5x - 75 = 0)$ and solving $(x = \frac{15}{2}, -5)$ If the -5 is quoted in the answer without being discarded (placing in brackets is sufficient to show non-inclusion) then the A mark at the end is lost.]

c) $\frac{24m^9}{64m^6} = 81$ giving $\frac{3}{8}m^3 = 81$ Adds and subtracts indices at any stage **[M1]**

Handles powers correctly on bottom line **[M1]**

$m = 6$ **[A1]**

d) i. Uses cosine formula $[21^2 = 16^2 + 18^2 - 2 \times 16 \times 18 \times \cos P]$ **[M1]**

Calculates correctly in the right order **[M1]**

Angle P = anything rounding to 76 (degrees) or 1.33 radians **[A1]**

ii. Uses sine formula $[\frac{\sin Q}{18} = \frac{\sin \text{their } P}{21}]$ **[M1]**

Anything rounding to 56.3 (degrees) or 0.982 radians **[A1]**

iii. $16 \sin \text{their } Q$ $(= 13.3)$ (metres) **[A1ft]**

Question B3

a) i. $2y + 8x = 88$ **[M1]**

$y = \frac{88 - 8x}{2}$ or equivalent **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$A = 3x \times y + x^2$ **[M1*]**

Substitutes in their y **[M1*]**

Reaches result with no errors seen and with both M marks scored. **[A1]**

iii. Attempts to differentiate (sight of 132 or an x term is sufficient for this mark) ($\frac{dA}{dx} = 132 - 22x$) **[M1*]**

Sets equal to 0 and solves **[M1]**

$x = 6$ **[A1]**

iv. Attempts to find $\frac{d^2A}{dx^2}$ (sight of a constant term is sufficient for this mark) **[M1*]**

$= -22$ correct answer **[A1]**

This is negative, so there is a maximum (Allow follow through for their $\frac{d^2A}{dx^2}$ provided it is negative. **[A1ft]**

or takes a numerical value between 0 and 6 and shows $\frac{dA}{dx} > 0$ **(M1*)**

takes a numerical value above 6 and shows $\frac{dA}{dx} < 0$ **(M1*)**

Thus there is a maximum (allow follow through for their $\frac{dA}{dx}$ provided the same outcome is obtained) **(A1ft)**

Part b) is on the next page.

Question B3 – (continued)

b) i. Differentiates (sight of an x term is sufficient for this mark) $(x-4)$ **[M1*]**

Substitutes $x=2$ into their differentiated expression (-2), inverts and changes sign ($\frac{1}{2}$) **[M1]**

$y = \frac{1}{2}x + 2$ (must be in this form) **[A1]**

ii. Finds where line l crosses the $y-$ axis $(0,\ 2)$ **[M1]**

Finds area under line $(=5)$ **[M1]**

Attempts to find $\int_2^4 \left(\frac{1}{2}x^2 - 4x + 9\right)\,dx$ (sight of x^3, x^2 or x is sufficient for this mark ($\frac{1}{6}x^3 - 2x^2 + 9x$) **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round $(14\frac{2}{3} - 11\frac{1}{3} = \frac{10}{3})$ **[M1]**

Adds areas **[M1]**

$= \frac{25}{3}$ or equivalent, or anything rounding to 8.33 **[A1]**

Question B4

a) i. $f(x) \geq 4$ (Accept $y \geq 4$ but $x \geq 4$ is B0) **[B1]**

ii. Rearranges $y = e^{2x} + 3$ and exchanges x and y at some stage **[M1]**

$f^{-1}(x) = \frac{1}{2}\ln(x-3)$ or equivalent **[A1]**

iii. $f'(x) \geq 0$ (Again, allow $y \geq 0$ but not $x \geq 0$) **[B1]**

iv. Combines functions in the correct order $[\frac{3(e^{2x}+3)-6}{2}]$ **[M1]**

Sets equal to 15 and reaches $e^{2x} = \cdots (9)$ **[M1]**

$x = \frac{1}{2}\ln 9$ or $\ln 3$ **[A1]**

b) i. Any odd and periodic function quoted (e.g. $\sin x, \tan x$, etc.) **[B1]**

ii. Any even function quoted (e.g. $\cos x$, $|x|$, x^2, etc.) **[B1]**

iii. $2x + 5 = \pm 11$ or squares both sides and reaches $2x = \pm\sqrt{121} - 5$ or $(x+8)(x-3) = 0$ **[M1]**

$x = 3$ **(A1)** $x = -8$ **(A1)** **[A2]**

c) i. $\frac{5}{3}$ **[B1]**

ii. Some valid method seen, such as $1 - \left(\frac{3}{5}\right)^2$, and reaching $\frac{4}{5}$. **[M1*]**

iii. $\frac{3}{4}$ **[B1]**

iv. Some valid method seen, such as $2 \times \frac{4}{5} \times \frac{3}{5}$, and reaching $\frac{24}{25}$. **[M1*]**

d) i. Expands $\cos(60-45)$ or $\cos(45-30)$ **[M1*]**

Replaces each term with a surd ($\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$) **[M1*]**

$\frac{1+\sqrt{3}}{2\sqrt{2}}$ or equivalent but it must be in exact form. **[A1]**

ii. Quotes answer from previous part. **[A1ft]**

Question B5

a) i. $12 + 3\lambda = -13 + 4\mu$; $-2 - \lambda = 21 - 5\mu$; $19 + 4\lambda = 15 - 2\mu$ **[M1]**

Takes any two equations and finds one of the unknowns **[M1]**

Finds second unknown **[M1]**

$\lambda = -3, \quad \mu = 4$ **[M1]**

Confirms the scalars satisfy the third equation **[M1*]**

A lies at $(3,\ 1,\ 7)$ **[A1]**

ii. Finds $(3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}).(4\boldsymbol{i} - 5\boldsymbol{j} - 2\boldsymbol{k})$ $(= 9)$ **[M1]**

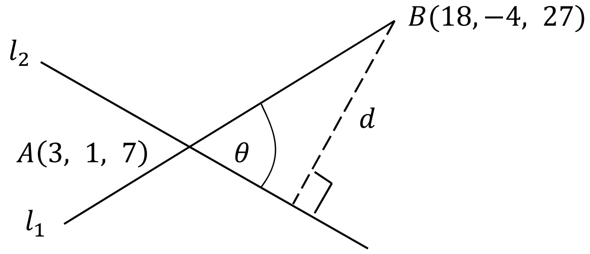
Finds magnitude of each vector $(\sqrt{26},\ \sqrt{45})$ and applies **[M1]**

$\cos\theta = \frac{\text{scalar product}}{\text{product of magnitudes}}$ $\left(\frac{9}{\sqrt{(26 \times 45)}}\right)$

Anything rounding to 74.7 (degrees) or 1.30 radians. **[A1]**

iii. Sets $12 + 3\lambda = 18$; $-2 - \lambda = -4$; $19 + 4\lambda = 27$ and shows that $\lambda = 2$ for all three equations. **[M1*]**

iv.



Finds the length of AB **[M1]**

$d = \text{their } AB \sin \text{their } \theta \quad (\sqrt{650} \times \sin 74.7)$ **[M1]**

= anything rounding to 24.6 or 24.5 **[A1]**

Parts b) and c) are on the next page.

Question B5 – (continued)

b) Uses Quotient Rule $[\left(\frac{dy}{dx} = \right)\frac{2x(x + 4) - (x^2 - 15)}{(x + 4)^2}$ **[M1*]**

Sets top line equal to 0 and forms a quadratic equation $(x^2 + 8x + 15 = 0)$ **[M1]**

Factorises or uses formula $[(x+3)(x+5) = 0 \text{ or } x = \frac{-8 \pm\sqrt{[8^2 - 4 \times 1 \times 15]}}{2 \times 1}]$ **[M1]**

Substitutes their values of x $(-3, -5)$ into original expression. **[M1]**

$(-3, -6)$; $(-5, -10)$ **[A1]**

Special case: if the Product Rule is used, the M1* mark is lost and the candidate must arrive at the correct quadratic equation for the next M mark. The following M1, M1 and A1 marks are then possible. [Maximum 4 out of 5].

c) $4^x \ln 4$ or $4(4^x)^3$ seen **[M1]**

$\left(\frac{dy}{dx} = \right) 4(4^x \ln 4)(4^x)^3$ **[A1]**

Allow any correct equivalent form, but this mark is lost if the candidate then attempts to simplify and makes an error.

Question B6

a) i. Uses $\sin^2\theta + \cos^2\theta = 1$ and divides by $\cos^2\theta$. **[M1*]**

Obtains result with no errors seen and the M mark scored. **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$dx = \sec^2\theta \, d\theta$ or $\frac{1}{\cos^2\theta} \, d\theta$ **[M1*]**

Writes integral in terms of θ ($\int \frac{1}{1 + \tan^2\theta} \times \sec^2\theta \; d\theta$) **[M1*]**

Uses result from part i ($\int \frac{\sec^2\theta}{\sec^2\theta} \; d\theta$) **[M1*]**

$= \int 1 \; d\theta = \theta \, (+c) = \tan^{-1}x + c$ (Each step must be seen) **[A1]**

iii. Separates variables ($y \, dy = \frac{1}{1 + x^2} \; dx$) **[M1*]**

Integrates both sides with a constant on one side ($\frac{y^2}{2} = \tan^{-1}x + c$) **[M1*]**
[If the constant is missed, this mark and all subsequent marks are lost.]

Substitutes boundary conditions into their expression and finds a value for c $(= 0)$ **[M1]**

$y = \sqrt{(2 \tan^{-1}x)}$ (but do not accept if $c = 0$ has not been established). **[A1]**

b) Area $\approx \frac{0.5}{3}$ **(M1)** $[(1 + 2.236) + 4(1.118 + 1.803) + 2(1.414)]$ **(M1)** **[M2]**

$=$ anything rounding to 2.96 **[A1]**

c) $5x + 2 = A(x + 1) + B(2x - 4)$ **[M1]**

$A = 4$ **(A1)** $B = \frac{1}{2}$ or equivalent **(A1)** **[A2]**

d) Volume $= \pi \int_0^4 (x^3 + 4) \, dx$ (Uses correct formula) **[M1*]**

Attempts to integrate (sight of x^4 or x is sufficient for this mark)
$[\, \pi \left(\frac{1}{4}x^4 + 4x \right)]$ [This mark is not lost if the π is dropped) **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 80\pi$ or anything rounding to 251 [If the correct answer appears with no working, award 1 mark out of 4.] **[A1]**

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