NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination
2017-18

Examination Session	Time Allowed
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Line l passes through the points $(0,\ 8)$ and $(10,\ 13)$.

a) Find the equation of line l. Give your answer in the form $y = mx + c$. **[3]**

Line l crosses the $x-$ axis at the point $(p,\ 0)$.

b) Find the value of p. **[1]**

Question A2

Three letters are selected, one after the other with no replacement, from the word

$$RAZZAMATAZZ$$

Find the probability that **none** of the letters selected is $'Z'$. **[3]**

Question A3

When $8x^3 - 12x^2 + 4x + k$ is divided by $(2x - 1)$, the remainder is 1.

Use the Remainder Theorem to find the value of k. **[3]**

Question A4

The 2nd term of a geometric series is 3000 and the 7th term is 7.29

Find the first term and the common difference. **[5]**

Question A5

Solve the equation

$$8^x = 2700$$

Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[3]**

Question A6

Solve the equation $\sin 2\theta = 0.53 \qquad (0° \leq \theta \leq 360°)$ **[5]**

Question A7

Find the equation of the tangent to the curve $y = e^{2x} + 6$ when $x = \ln 3$. **[4]**

Question A8

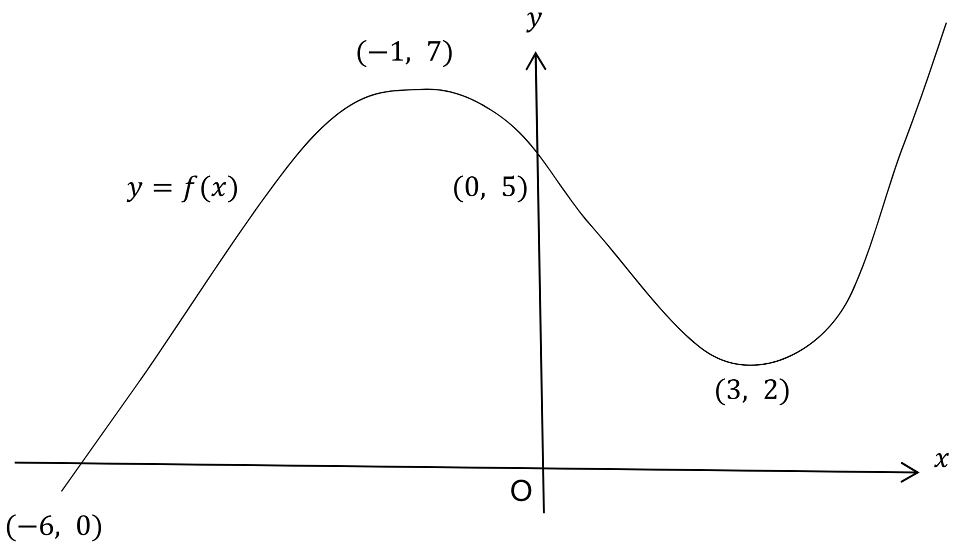

Figure 1

Figure 1 shows the graph of $y = f(x)$. The curve crosses the $x -$ axis at $(-6,\ 0)$ and crosses the $y -$ axis at $(0,\ 5)$. There are stationary values at $(-1,\ 7)$ and $(3,\ 2)$.

Sketch the curve with equation $y = f(-x)$. On your sketch, show clearly the coordinates where the curve crosses the $x -$ and $y -$ axes; and also the coordinates of any stationary values. **[3]**

Question A9

A vending machine dispenses quantities of coffee with a mean volume of 290 ml and standard deviation 5 ml. The quantities of coffee can be assumed to follow a Normal distribution.

The cups to be filled with coffee have a volume of 300 ml.

About what percentage of cups will be overfilled? **[3]**

Question A10

A curve has equation $4x^2 + x^3y - \frac{1}{2}y^4 = 7$

Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

Question A11

Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined as $\boldsymbol{a} = -3\boldsymbol{i} + p\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = 5\boldsymbol{i} - 3\boldsymbol{j} + 2p\boldsymbol{k}$.

The vectors are perpendicular to each other.

Find the value of p. **[3]**

Question A12

Use integration by parts to find

$$\int 128x^2\, e^{4x}\ dx.$$

[5]

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) Solve $2x^2 - x \leq 0$. **[3]**

b) The function $f(x)$ is defined as $f(x) = x^3 - 3x^2 - 4x + 12$.

i. Divide $f(x)$ by $(x - 2)$. **[3]**

ii. Factorise $f(x)$ completely. **[1]**

iii. Solve $f(x) = 0$. **[1]**

iv. Sketch the graph of $y = f(x)$. Show clearly where your curve crosses the $x -$ axis and the $y -$ axis. You do **not** need to show the coordinates of the stationary values. **[3]**

c) The 10th term of an arithmetic series is 3 times larger than the 3rd term.

i. Show that, if a is the first term and d is the common difference, **[2]**

$$a = \frac{3}{2}d$$

The sum of the first 40 terms is 5040.

ii. Find the first term and the common difference. **[5]**

d) Find the coefficient of the term in x^4 in the expansion of $(3 - 2x)^7$. **[2]**

Question B2

a) A survey is carried out on the number of people who live in a town. The number of people, N, after t years from when the survey started is given by the formula

$$N = 80000\, e^{0.015t} \quad (0 \le t \le 15)$$

i. State the number of people in the town when the survey began. **[1]**

ii. Find the number of people after 5 years. **[2]**

iii. Find $\frac{dN}{dt}$. **[1]**

iv. Find the value of t when the number of people is increasing at a rate of 1313 people per year. **[3]**

b) Solve the equation **[4]**

$$\log_3(x^2 + 5x) - 1 = \log_3(x^2 - 25) \qquad (x > 5)$$

c) Find the value of m if **[3]**

$$\frac{3m^5 \times 8m^4}{(4m^2)^3} = 81.$$

d)

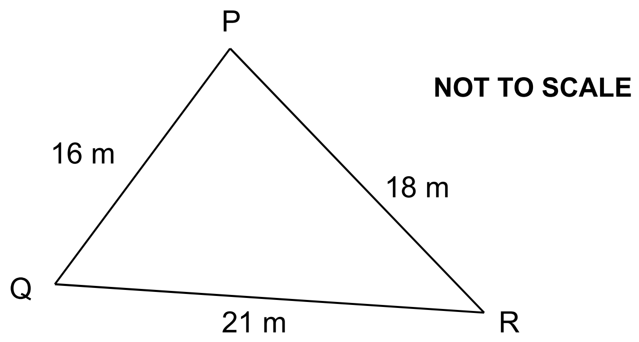

Figure 2

Figure 2 shows acute-angled triangle PQR with PQ = 16 m, PR = 18 m and QR = 21 m.

i. Find angle P. **[3]**

ii. Find angle Q. **[2]**

iii. Find the shortest distance from point P to line QR. **[1]**

Question B3

a)

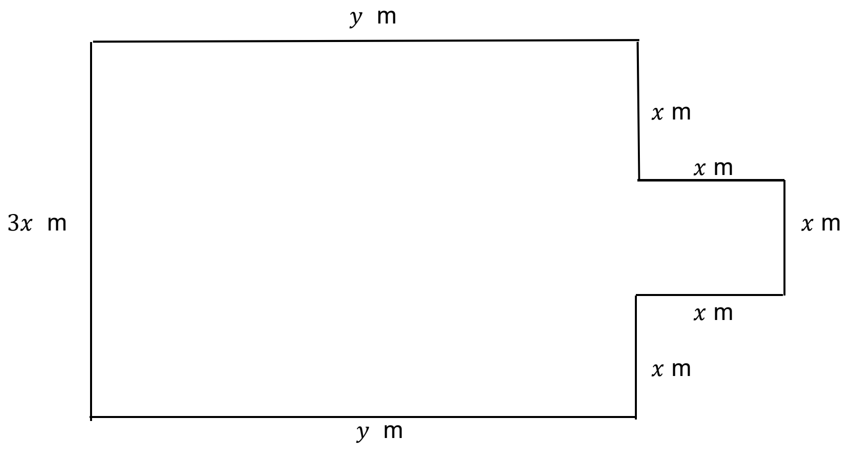

Figure 3

Figure 3 shows a lawn which has dimensions as shown. The perimeter of the lawn is 88 metres.

i. Express y in terms of x. **[2]**

ii. Show that the area of the lawn, A, is given by

$$A = 132x - 11x^2$$ **[3]**

iii. Use $\frac{dA}{dx}$ to find the value of x which gives the maximum area. **[3]**

iv. Confirm that your value of x gives a maximum. **[3]**

Part b) is on the next page.

Question B3 – (continued)

b)

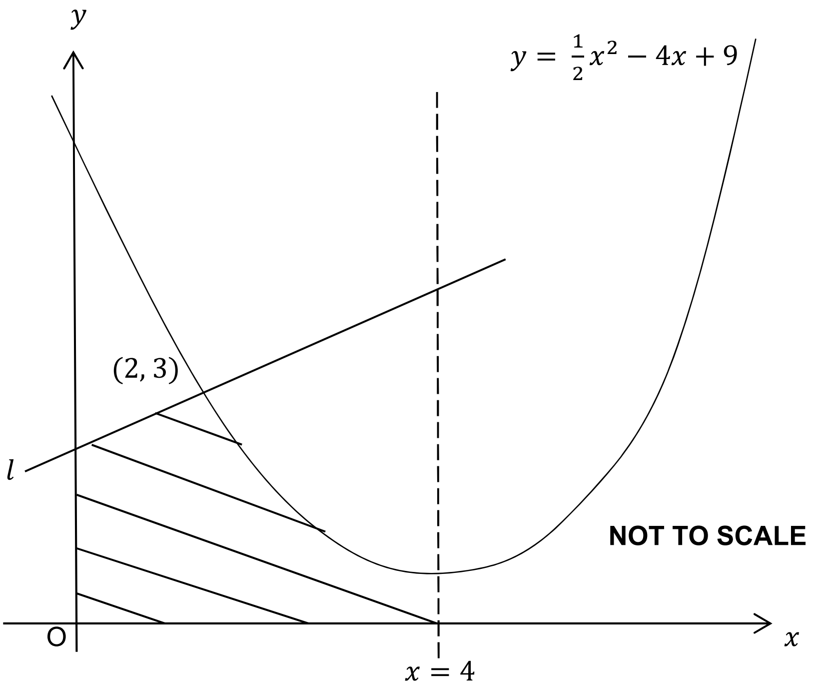

Figure 4

Figure 4 shows the curve $y = \frac{1}{2}x^2 - 4x + 9$ and line l which is a normal to the curve at the point $(2,\ 3)$. The line $x = 4$ is also shown.

i. Find the equation of line l. Give your answer in the form $y = mx + c$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by line l, the line $x = 4$, the curve $y = \frac{1}{2}x^2 - 4x + 9$, and the $x -$ and $y -$ axes. **[6]**

Question B4

a) The function $f(x)$ is defined as $f(x) = e^{2x} + 3 \quad (x \geq 0)$

The function $g(x)$ is defined as $g(x) = \frac{3x - 6}{2} \qquad (-\infty < x < \infty)$

i. State the range of $f(x)$. **[1]**

ii. Find $f^{-1}(x)$ in terms of x. **[2]**

iii. State the range of $f^{-1}(x)$. **[1]**

iv. Solve the equation $g(f(x)) = 15$. Give your answer as an exact logarithm. **[3]**

b) i. Give an example of an *odd* function which is also *periodic*. **[1]**

ii. Give an example of an *even* function. **[1]**

iii. Solve $|2x + 5| = 11$ **[3]**

c) Angle A is acute and $\cos A = \frac{3}{5}$.

i. Write down the value of $\sec A$. Give your answer in the form $\frac{m}{n}$ where m and n are integers. **[1]**

ii. Show that $\sin A = \frac{4}{5}$. *All working must be shown*. **[1]**

iii. Write down the value of $\cot A$. Give your answer in the form $\frac{m}{n}$ where m and n are integers. **[1]**

iv. Show that $\sin 2A = \frac{24}{25}$. *All working must be shown*. **[1]**

d) i. You are given $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60 = \frac{1}{2}$ and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

Find the **exact** value of $\cos 15°$. *All working must be shown*. **[3]**

ii. Hence state the value of $\sin 75°$. **[1]**

Question B5

a) Line l_1 has equation $\boldsymbol{r} = (12\boldsymbol{i} - 2\boldsymbol{j} + 19\boldsymbol{k}) + \lambda(3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})$ where λ is a scalar.

Line l_2 has equation $\boldsymbol{r} = (-13\boldsymbol{i} + 21\boldsymbol{j} + 15\boldsymbol{k}) + \mu(4\boldsymbol{i} - 5\boldsymbol{j} - 2\boldsymbol{k})$ where μ is a scalar.

i. Show that lines l_1 and l_2 intersect,and find the coordinates of A which is their point of intersection. **[6]**

ii. Find the acute angle between lines l_1 and l_2. **[3]**

iii. Show that point $B(18,\ -4,\ 27)$ lies on line l_1. **[1]**

iv. Find the shortest distance from point B to line l_2. **[3]**

b) A curve has equation

$$y = \frac{x^2 - 15}{x + 4}$$

Use the Quotient Rule to find $\frac{dy}{dx}$ and hence find the coordinates of the stationary values on the curve. **[5]**

c) Differentiate $(4^x)^4$. **[2]**

Question B6

a) i. By using a suitable trigonometric formula, show that $\tan^2\theta + 1 = \sec^2\theta$. **[2]**

ii. By using the substitution $x = \tan\theta$, show that

$$\int \frac{1}{1+x^2}\ dx = \tan^{-1}x + constant$$ **[4]**

Each stage of your working must be clearly shown.

iii. Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{y(1+x^2)} \qquad (-\frac{\pi}{2} < x < \frac{\pi}{2})$$

subject to $y = 0$ when $x = 0$. **[4]**

b) The table below shows the values of $\sqrt{(x^2+1)}$ given to 3 decimal places where appropriate for $x = 0,\ 0.5,\ 1,\ 1.5$ and 2.

$\boldsymbol{x}$	0	0.5	1	1.5	2
$\boldsymbol{\sqrt{(x^2+1)}}$	1	1.118	1.414	1.803	2.236

Use Simpson's Rule with four intervals to find an estimate of

$$\int_0^2 \sqrt{(x^2+1)}\, dx.$$ **[3]**

c) Express $\frac{5x+2}{(2x-4)(x+1)}$ in the form $\frac{A}{2x-4} + \frac{B}{x+1}$ where A and B are constants to be determined. **[3]**

d) The curve $y = \sqrt{(x^3+4)}$ is rotated about the $x-$ axis between $x = 0$ and $x = 4$.

Find the volume formed. **[4]**

This is the end of the examination.

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