THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A9. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Finds gradient of BC $[\frac{k\ -\ 16}{14\ -\ -\ 2}]$ **[M1]**

Finds gradient of AB $(= 2)$ **[M1]**

Inverts $(-\frac{1}{2})$ and sets equal to their gradient of BC **[M1]**

Rearranges and solves **[M1]**

$k = 8$ **[A1]**

Question A2

$2c - 1 + 8c - 4 = 1$ **[M1]**

$c = \frac{3}{5}$ or equivalent **[A1]**

$p(X) = \frac{1}{5}$ (Allow follow through provided $0\ \leq \text{their}\ p(X)\ \leq 1$) **[A1ft]**

Question A3

Substitutes $x = -2$ into expression (Remainder Theorem must be used) **[M1*]**

Sets equal to $3p$ and finds a value for p **[M1]**

$p = 3$ **[A1]**

Question A4

$^7C_5 \times (\frac{1}{q})^2 \times q^5$ **(M1)** $^7C_4 \times (\frac{1}{q})^3 \times q^4$ **(M1)** (Allow ^xC_y for ^yC_x and the presence of x) **[M2]**

Multiplies second expression by 15 or divides first by 15, sets equal to each other and reaches $q^2 = \cdots\ (25)$ **[M1]**

$q = \pm 5$ **[A1]**

Question A5

Multiplies through $[\,e^{2x} - 10 = e^{x} + 10\,]$ **[M1*]**

Recognises the 'hidden quadratic equation' $[e^{2x} - e^{x} - 20 = 0]$ **[M1*]**

Factorises or uses formula $[(e^{x} - 5)(e^{x} + 4) = 0$ or $e^{x} = \frac{1 \pm\sqrt{[(-1)^2 - 4 \times 1 \times -20]}}{2 \times 1}]$ **[M1]**

$x = \ln 5$ (If the candidate provides an answer to $e^{x} + 4 = 0$, this mark is lost.) **[A1]**

Question A6

$\frac{1}{2}(a-3)(2a+3)\sin 30 = 7a + 6$ **[M1]**

Multiplies out and forms a quadratic equation $[2a^2 - 31a - 33 = 0]$ **[M1]**

Factorises or uses formula $[(2a - 33)(a + 1) = 0$ or $a = \frac{31 \pm\sqrt{[(-31)^2 - 4 \times 2 \times -33]}}{2 \times 2}]$ **[M1]**

$a = 16\frac{1}{2}$ or equivalent (Ignore any reference to the -1) **[A1]**

Question A7

Multiplies out integrand $[\,x^2 + 2 + \frac{1}{x^2}\,]$ **[M1*]**

Attempts to integrate (sight of x^3, x or reciprocal x is sufficient for this mark) **[M1*]**

$\frac{x^3}{3} + 2x - \frac{1}{x} + c$ **(A1)** for any 2 correct; **(A2)** for all correct and $+\,c$ **[A2]**

Question A8

$3x - 4 = 14$ or $3x - 4 = -14$ or similar **[M1]**

$x = 6$ **(A1)** $x = -\frac{10}{3}$ or equivalent, or anything rounding to -3.33 **(A1)** **[A2]**

or for **(M1)** squares through $[(3x-4)^2 = 196]$ and rearranges

Question A9

$f'(x) = 5x^4$ **[M1*]**

Applies correct formula $[\,3 - \frac{3^5\ -\ 230}{\text{their } f'(3)}\,]$ **[M1*]**

= 2.9679… (can be implied) **[A1]**

= 2.968 to 4 significant figures (allow follow through) **[A1ft]**

Question A10

$z = \frac{42\ -\ 40}{2}\ \ (= 1)$ **[M1]**

Finds Φ(1) (= 0.8413) **[M1]**

Subtracts their ϕ(1) from 1 and multiplies by 70 **[M1]**

Anything rounding to 11 days. (Allow follow through) **[A1ft]**

Question A11

Please note: this is a 'show that' question so all working must be seen.

$\left(\frac{dy}{dx} =\right) \frac{\cos x \times 0 -\ [1] \times (-\sin x)}{\cos^2 x}$ (Allow if the negative sign in front of $\sin x$ is missing) **[M1*]**

$\frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ (this must be seen, and it must be correct with no sign errors) **[M1*]**

$= \sec x \tan x$ (No errors seen and both M marks scored) **[A1]**

Question A12

$2x + 19 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ **[M1]**

$A = 1$ **[A1]**

$B = \ -1$ **[A1]**

$C = \ -3$ **[A1]**

Section B

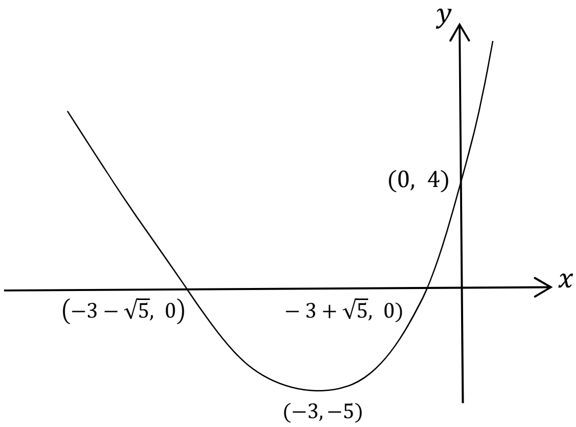
Question B1

a) i. $(x+3)^2$ **(B1)** -5 **(B1)** **[B2]**

ii. Solves and reaches $x + 3 = \pm\sqrt{\ldots}$ **[M1*]**

$x = -3 + \sqrt{5}$ or $-3 - \sqrt{5}$; or $-3 \pm \sqrt{5}$ **[A1]**

iii.



Correct shape **[B1]**

$(-3 \pm \sqrt{5}, 0)$ shown **[B1]**

$(-3,-5)$; $(0,4)$ shown **[B1]**

b) Solves to find two critical values $(0,-9)$ **[M1]**

$x \le -9$ **(A1)** $x \ge 0$ **(A1)** **[A2]**
Please note: the two ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen.

c) i. $a + 10d = \frac{1}{2}a$ **[M1*]**

At least one intermediate line of working followed by $a = -20d$ with no errors seen. **[A1]**

ii. $1020 = \frac{51}{2}[2 \times -20d + (51-1)d]$ or $1020 = \frac{51}{2}[2a + (51-1) \times -\frac{a}{20}]$ **[M1]**

Calculates correctly in the right order **[M1]**

$d = 4$ **(A1)** $a = -80$ **(A1)** **[A2]**

d) i. $\dfrac{a[1-(\frac{3}{5})^7]}{1-\frac{3}{5}} = 37969$ **[M1]**

Rearranges correctly **[M1]**

$a = 15625$ (Full answer here and in part ii required – no rounding off) **[A1]**

ii. $[\left(15625 \div \left(-\frac{3}{5}\right)\right] = 39062\frac{1}{2}$ or equivalent (Allow follow through) **[A1ft]**

Question B2

a) i. Substitutes $x = 0.1$ into formula **[M1]**

Anything rounding to 390 **[A1]**

ii. Rearranges and reaches $e^{0.6x} = \cdots (\frac{648}{480})$ **[M1]**

Takes logs and reaches $0.6x = \cdots [\ln\left(\frac{648}{480}\right)]$ **[M1]**

Anything rounding to 0.5 **[A1]**

iii. $\frac{dy}{dx} = 480 \times 0.6 \times e^{0.6x}$ **[M1]**

Substitutes $x = 1.4$ into their $\frac{dy}{dx}$ **[M1]**

Anything rounding to 667 **[A1]**

b) $\log_4\left[\frac{(x+2)(x+5)}{(x+2)(x-1)}\right] = \log_4 4$

Uses log subtraction law to combine terms on LHS **[M1*]**

Adapts RHS and removes logs at the right time **[M1*]**

Solves $[\frac{x+5}{x-1} = 4]$ **[M1]**

$x = 3$ **[A1]**

If the cancellation is missed, the third M mark can be given for forming a quadratic equation $[3(x^2 - x - 6) = 0]$ and solving $(x = 3,\ -2)$. If the -2 is then not discarded (putting it in brackets is sufficient) the A mark is lost.

c) i. Uses cosine formula $[1744 = 54^2 + 58^2 - 2 \times 54 \times 58 \times \cos\theta]$ **[M1]**

Calculates correctly in the right order **[M1]**

$\cos\theta = \frac{21}{29}$ [Please note: candidates who work backwards from part ii score no marks] **[A1]**

ii. Uses $\cos^2\theta + \sin^2\theta = 1$ or a right-angled triangle, or any other valid method **[M1*]**

Reaches $\sin\theta = \frac{20}{29}$ with no errors seen and the M mark scored. **[A1]**

iii. Uses sine formula $[\frac{\sin B}{58} = \frac{20/29}{\sqrt{1744}}]$ **[M1]**

Anything rounding to 73.3 (degrees) **[A1]**

iv. 40 (cm) **[B1]**

Question B3

a) i. $16xh + 15x^2 = 2880$ **[M1]**

$h = \frac{2880\ -\ 15x^2}{16x}$ or equivalent such as $\frac{180}{x} - \frac{15x}{16}$ **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$V = 15x^2h$ **[M1*]**

Substitutes their h into formula for volume **[M1*]**

Reaches $V = 2700x - \frac{225x^3}{16}$ with no errors seen and both M marks scored. **[A1]**

iii. Attempts to differentiate (sight of 2700 or x^2 is sufficient for this mark) $[\frac{dV}{dx} = 2700 - \frac{675}{16}x^2\ ]$ **[M1*]**

Sets equal to 0 **[M1]**

Reaches $x^2 = \cdots\ \ (64)$ **[M1]**

$x\ = 8$ (or -8) [If the -8 is not discarded (placing in brackets is sufficient to show non-inclusion), this mark is lost.] **[A1]**

iv. Differentiates a second time (sight of x term is sufficient for this mark) **[M1*]**

$\frac{d^2V}{dx^2} = -\frac{1350x}{16}$ (correct answer) **[A1]**

This is negative (when $x = 8$) so there is a maximum Allow follow through for their $\frac{d^2V}{dx^2}$ provided it gives a maximum **[A1ft]**

or Takes a numerical value between 0 and 8 and shows $\frac{dV}{dx} > 0$ **(M1*)**

Takes a numerical value above 8 and shows $\frac{dV}{dx} < 0$ **(M1*)**

Thus there is a maximum (at $x = 8$) **(A1ft)** Allow follow through for their $\frac{dV}{dx}$ provided it gives a maximum

Part b) is on the next page.

Question B3 – (continued)

b) i. Finds $\frac{dy}{dx}$ $(= -\frac{1}{2}x)$ and substitutes $x = 4$ $(= -2)$ **[M1*]**

Inverts and changes sign $(= \frac{1}{2})$ **[M1]**

$y = \frac{1}{2}x + 3$ **[A1]**

ii. Finds area under line $l = \frac{1}{2}(3 + 5) \times 4$ $(= 16)$ **or** integrates their equation of line l between 0 and 4. **[M1]**

Attempts to integrate for the area under the curve (sight of x or x^3 is sufficient for this mark) [$9x - \frac{1}{12}x^3$] **[M1*]**

[M1]

Substitutes limits into their integrated expression and subtracts the right way round $(36 - 30\frac{2}{3} = 5\frac{1}{3})$ **[M1]**

Adds areas **[A1]**

$= \frac{64}{3}$ or equivalent, or anything rounding to 21.3

Question B4

a) i. Yes **(B1*)** with a reason **(B1)** such as 'every element in the domain is mapped on to a unique element in the range' or similar words.
*Only available if a reason, even a wrong one, follows. **[B2]**

ii. $f(x) < \frac{1}{4}$. Allow $y < \frac{1}{4}$ but $x < \frac{1}{4}$ is B0 **[B1]**

iii. Shows an understanding of inverse $[\, g^{-1}(x) = \frac{3x-1}{16}]$ and substitutes in 11 <u>or</u> solves $g(x) = 11$ **[M1]**

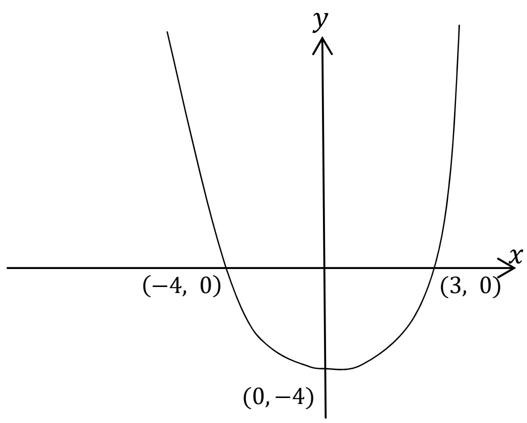
$g^{-1}(11) = 2$ **[A1]**

iv. Applies the functions in the correct order $\left\{\left[16\left(\frac{1}{e^x+3}\right)+1\right] \div 3 (= 2)\right\}$ **[M1]**

Sets equal to 2 and solves **[M1]**

$x = \ln(\frac{1}{5})$ or $-\ln 5$ **[A1]**

b)



Correct shape **[B1]**

(-4, 0) and $(3, 0)$ shown **[B1]**

$(0,$ -4$)$ shown **[B1]**

Part c) is on the next page.

Question B4 – (continued)

c) i. Uses addition cosine formula with both angles set equal to each other **[M1*]**
$[\cos(\theta + \theta) = \cos\theta\cos\theta - \sin\theta\sin\theta]$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ M mark scored and no errors seen. **[A1]**

ii. Replaces $\cos^2\theta$ by $1 - \sin^2\theta$ in previous result **[M1*]**

Reaches $\cos 2\theta = 1 - 2\sin^2\theta$ with no errors seen **[A1]**

ii. Replaces $\cos 2\theta$ by $1 - 2\sin^2\theta$ and forms a quadratic equation in $\sin\theta$ **[M1]**
$[6\sin^2\theta + \sin\theta - 2 = 0]$

Factorises or uses formula
$[(2\sin\theta - 1)(3\sin\theta + 2) = 0 \text{ or } \sin\theta = \frac{1 \pm \sqrt{[(-1)^2 - 4 \times 6 \times -2]}}{2 \times 6}]$ **[M1]**

$\sin\theta = \frac{1}{2}, -\frac{2}{3}$ (can be implied) **[A1]**

$\theta = 30,\ 150;$ and anything rounding to $222;\ 318$ (degrees) **[A2]**

(A1) for any two correct; **(A2)** for all correct.

If extra solutions appear that are in the range, one A mark is lost. Ignore solutions outside the range.

Question B5

a) i. $[\boldsymbol{r} =](-3\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k})$ or $(9\boldsymbol{i} - 4\boldsymbol{j} + 22\boldsymbol{k}) + t(12\boldsymbol{i} - 6\boldsymbol{j} + 18\boldsymbol{k})$ Allow any other correct form [e.g. $[\boldsymbol{r} =]\ (9\boldsymbol{i} - 4\boldsymbol{j} + 22\boldsymbol{k}) + \mu(2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k})]$.
(B1) for first part; **(B1)** for directional vector with a scalar **[B2]**

ii. Finds $\overrightarrow{AC}(9\boldsymbol{j} + 17\boldsymbol{k})$ and $\overrightarrow{AB}(12\boldsymbol{i} - 6\boldsymbol{j} + 18\boldsymbol{k})$ and finds their scalar product $(=252)$ **[M1]**

Finds the magnitudes of $\overrightarrow{AC}$ and $\overrightarrow{AB}$ $(\sqrt{370},\ \sqrt{504})$ **[M1]**

$\cos\theta = \frac{\text{their scalar product}}{\text{product of their magnitudes}} \left(\frac{252}{\sqrt{370 \times 504}}\right)$ **[M1]**

$\theta =$ anything rounding to 54.3 (degrees) or 0.95 radians **[A1]**

iii. Finds the length of $BC\ (= \sqrt{370})$ **[M1*]**

$BC = AC$ but angle $\theta \neq 60°$ (**or** $AB = \sqrt{504}$ which is $\neq \sqrt{370}$) **[M1*]**

So triangle is isosceles but not equilateral. **[A1]**

iv. Sets $\boldsymbol{i}$ – components of vector equation of AB equal to 3;
$\boldsymbol{j}$ – components equal to -1 and $\boldsymbol{k}$ – components equal to 13. Shows that the scalar takes the same value for all three. **[M1*]**

v. Finds scalar product **[M1*]**

$= 0$ [There must be some evidence: just stating "$= 0$" is not enough] **[A1]**

They are perpendicular to each other (or similar words) **[A1]**

b) i. $8x - 2y - 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ Correct use of Product Rule $\left(\pm 2y \pm 2x\frac{dy}{dx}\right)$ **[M1*]**

Correct implicit differentiation ($\pm 2x\frac{dy}{dx}$ or $\pm 2y\frac{dy}{dx}$ seen) **[M1*]**

Gathers $\frac{dy}{dx}$ terms on to one side and factorises (only available if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{2y - 8x}{2y - 2x}$ or equivalent **[A1]**

ii. Sets top line of $\frac{dy}{dx}$ equal to 0 and reaches $y = 4x$. **[M1*]**

iii. Substitutes $y = 4x$ into their $\frac{dy}{dx}$ and finds at least one value for x **[M1]**
$(2,\ 8)$; $(-2, -8)$ **[A1]**

Question B6

a) i. $du = 3x^2\, dx$ or equivalent **[M1*]**

Writes integral in terms of u $\left(\int \frac{1}{3} \times \frac{1}{u}\ du\right)$ **[M1*]**

Attempts to integrate (sight of $\ln u$ is sufficient for this mark) and turns expression back into terms in x. **[M1]**

$\frac{1}{3}\ln(1+\ x^3) + c$ **[A1]**

ii. Separates variables $\left(\frac{1}{y}\ dy = \frac{x^2}{1\ +\ x^3}\ dx\right)$ **[M1*]**

Integrates both sides with a constant added to one side. (If the constant is missing, this mark and all subsequent ones are lost) **[M1*]**
$[\ln y = \frac{1}{3}\ln(1+\ x^3) + c]$

Substitutes in boundary conditions to find the constant ($= \ln 3$ if on the RHS) **[M1]**

$y = 3(1 + x^3)^{1/3}$ or equivalent but there must be no logarithms **[A1]**

b) Uses correct formula $[\pi\ \int_0^3 (4 -\ x^2)\ dx\ ]$ **[M1*]**

Attempts to integrate (sight of x or x^3 is sufficient for this mark) If the π is dropped, this mark is not lost $\pi[4x - \frac{x^3}{3}]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

$= 3\pi$ or anything rounding to 9.42 [If the correct answer appears with no working, award 1 mark out of 4.] **[A1]**

Parts c) and d) are on the next page.

Question B6 – (continued)

c) Uses integration by parts in the right direction **[M1*]**

$= 4x(-\cos x)$ **(A1) -** $\underline{\int_0^{\pi} 4(-\cos x)\ dx}$ **(A1)** for correct first part **[A1]**

$= -4x(\cos x) + \underline{4 \sin x}$ **(A1ft)** Follow through on second integration only (underlined parts) **[A1ft]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

[A1]

$= 4\pi$ or anything rounding to 12.6

d) $4(x^2 - 8x + 3)^3$ or $(2x - 8)$ seen **[M1*]**

$\frac{dy}{dx} = 4(2x - 8)\ (x^2 - 8x + 3)^3$ or equivalent **[A1]**

$= 0$ (when $x = 4$) [Allow follow through for their $\frac{dy}{dx}$] **[A1ft]**

Please note: if the candidate states "$= 0$" without any working, this scores no marks.

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