NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination
2017-18

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Point A lies at $(-8,\ 4)$, point B lies at $(-2,\ 16)$ and point C lies at $(14,\ k)$.

BC is perpendicular to AB.

Find the value of k. **[5]**

Question A2

The probability that event X occurs is $(2c-1)$. The probability that event X does not occur is $(8c-4)$. Events X and X' are mutually exclusive.

Find the value of c and hence find $p(X)$. **[3]**

Question A3

When $2x^3+(2p-1)x^2-px-1$ is divided by $(x+2)$, the remainder is $3p$.

Use the Remainder Theorem to find the value of p. **[3]**

Question A4

In the expansion of

$$(\frac{1}{q}+qx)^7 \quad (q \neq 0)$$

the term in x^5 is 15 times larger than the term in x^4.

Find the two values of q. **[4]**

Question A5

Solve the equation

$$\frac{e^{2x}-10}{e^x+10}=1.$$ **[4]**

Question A6

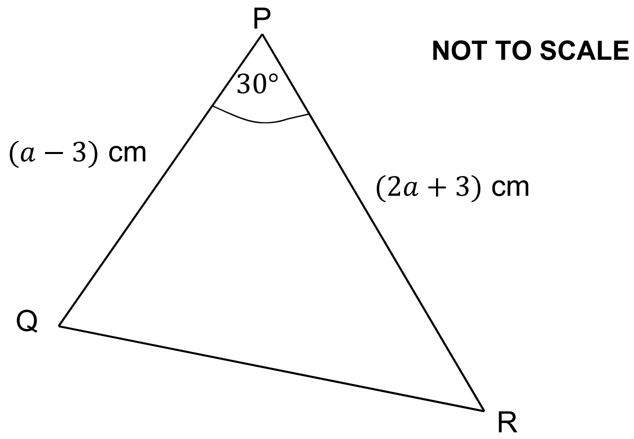

Figure 1

Figure 1 shows triangle PQR with PQ = $(a-3)$ cm, PR = $(2a+3)$ cm and angle P = $30°$.

The area of triangle PQR is $(7a+6)$ cm^2.

Find the value of a. **[4]**

Question A7

Find

$$\int (x + \frac{1}{x})^2 \ dx.$$

[4]

All working must be shown.

Question A8

Solve the equation

$$|3x-4| = 14.$$

[3]

Question A9

The function $f(x)$ is defined as $f(x) = x^5 - 230$.

Starting with $x = 3$, apply the Newton-Raphson method once to obtain a better approximation to the equation $f(x) = 0$.

Give your answer to **4** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[4]**

Question A10

The times taken for a large container to fill with water can be assumed to follow a Normal distribution with mean 40 minutes and standard deviation 2 minutes.

The container is filled every day over 70 days.

On approximately how many days will it take more than 42 minutes to fill the container? **[4]**

Question A11

By writing $\sec x$ as $\frac{1}{\cos x}$ and using the Quotient Rule, show that the differentiation of $\sec x$ is $\sec x \tan x$.

<u>Each</u> stage of your working must be clearly shown. **[3]**

Question A12

Write $\frac{2x + 19}{(x - 3)(x + 2)^2}$ in the form $\frac{A}{x - 3} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$ where A, B and C are constants to be determined. **[4]**

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a) i. Write $x^2 + 6x + 4$ in the form $(x + a)^2 + b$ where a and b are integers. **[2]**

ii. Hence solve the equation $x^2 + 6x + 4 = 0$ giving your answers in the form $p + \sqrt{q}$ and $p - \sqrt{q}$ where p and q are integers. **[2]**

iii. Sketch the graph of $y = x^2 + 6x + 4$, showing clearly the coordinates of any stationary value, and where the curve crosses the $x -$ axis and the $y -$ axis. **[3]**

b) Solve the inequality $x^2 + 9x \geq 0$. **[3]**

c) The 11^{th} term of an arithmetic series is half of the first term.

i. Show that if a is the first term and d is the common difference, then

$$a = -20d.$$ **[2]**

The sum of the first 51 terms is 1020.

ii. Find the value of d and the value of a. **[4]**

d) ***In this part, answers must be given <u>in full</u>, with <u>no</u> rounding off.***

A geometric series has common ratio $\frac{3}{5}$ and the sum of the first 7 terms is 37969.

i. Find the first term. **[3]**

ii. Find the sum to infinity. **[1]**

Question B2

a) Two variables, x and y, are connected by the formula

$$y = 480\,e^{0.6x} - 120.$$

i. Find y when $x = 0.1$ **[2]**

ii. Find x when $y = 528$ **[3]**

iii. Find $\frac{dy}{dx}$ when $x = 1.4$ **[3]**

b) Solve the equation

$$\log_4(x^2 + 7x + 10) - \log_4(x^2 + x - 2) = 1 \quad (x > 1)$$ **[4]**

All working must be shown.

c)

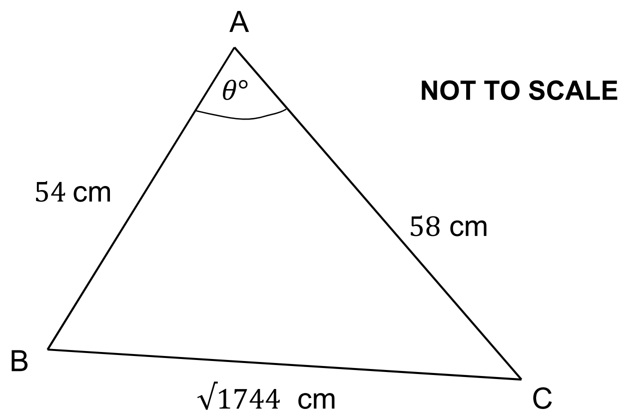

Figure 2

Figure 2 shows acute-angled triangle ABC with AB = 54 cm,
AC = 58 cm, BC = $\sqrt{1744}$ cm and angle A = $\theta°$.

i. Find $\cos\theta$, giving your answer in the form $\frac{m}{n}$ where m and n are integers. *All working must be shown*. **[3]**

ii. <u>Without</u> working out the size of θ, and showing <u>all</u> working, show that $\sin\theta = \frac{20}{29}$. **[2]**

iii. Find angle B. **[2]**

iv. Find the shortest distance from point C to side AB. **[1]**

Question B3

a)

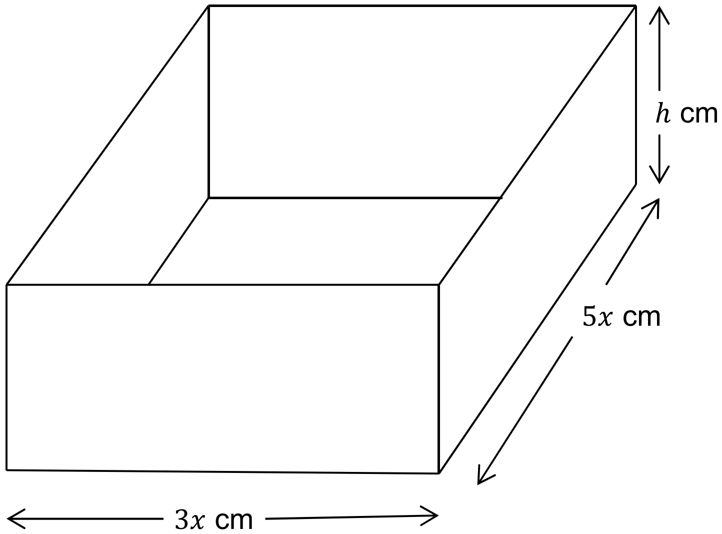

Figure 3

Figure 3 shows a box in the shape of a cuboid with length $5x$ cm, width $3x$ cm and height h cm.

The outside surface area is 2880 cm^2.

The box has a base **but it has no top.**

i. Express h in terms of x. **[2]**

ii. Show that the volume of the box, V, is given by

$$V = 2700x - \frac{225x^3}{16}$$

[3]

iii. Use $\frac{dV}{dx}$ to find the value of x which gives the maximum volume. **[4]**

iv. Confirm that your value of x gives a maximum. **[3]**

Part b) is on the next page.

Question B3 – (continued)

b)

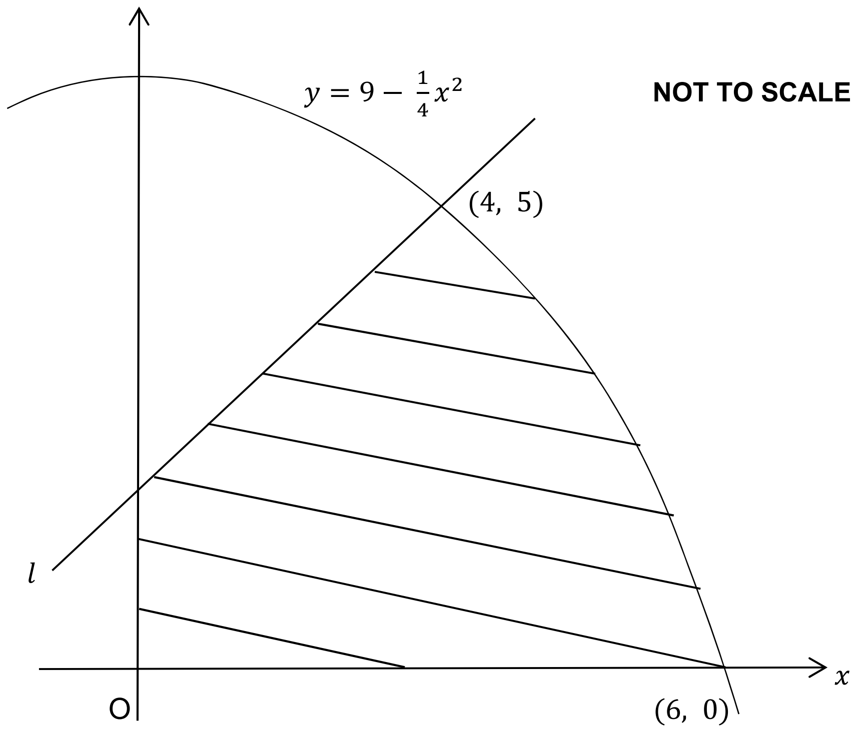

Figure 4

Figure 4 shows the curve $y = 9 - \frac{1}{4}x^2$ and the line l which is a normal to the curve at the point $(4,\ 5)$.

i. Find the equation of line l. Give your answer in the form $y = mx + c$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = 9 - \frac{1}{4}x^2$, line l and both the $x -$ and $y -$ axes. **[5]**

Question B4

a) Function $f(x)$ is defined as $f(x) = \frac{1}{e^x + 3}$ $(x > 0)$

Function $g(x)$ is defined as $g(x) = \frac{16x + 1}{3}$ $(-\infty < x < \infty)$

i. Is $g(x)$ a one-one function? Give a reason. **[2]**

ii. State the range of $f(x)$. **[1]**

iii. Find $g^{-1}(11)$. **[2]**

iv. Solve $g(f(x)) = 2$. Give your answer as an exact logarithm. **[3]**

b)

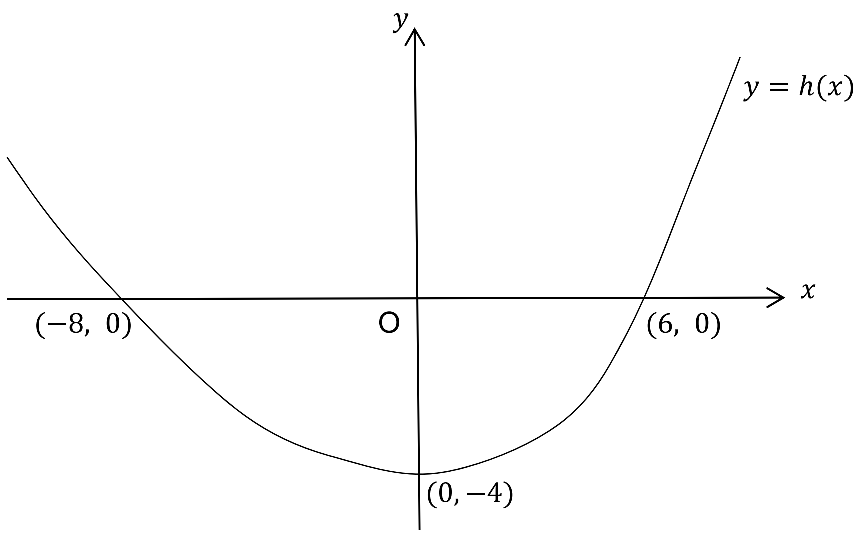

Figure 5

Figure 5 shows the function $y = h(x)$ which crosses the $x-$ axis at $(-8,\ 0)$ and $(6,\ 0)$; and has a stationary value at $(0, -4)$.

Draw a sketch of the graph $y = h(2x)$. On your sketch, show clearly the coordinates where the curve crosses the $x-$ axis, and also the coordinates of any stationary value. **[3]**

c) i. Use one of the addition formulae to prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$. **[2]**

ii. Show that $\cos 2\theta$ can also be written as $1 - 2\sin^2\theta$. **[2]**

iii. Solve the equation $\sin\theta - 3\cos 2\theta + 1 = 0$ $(0° \leq \theta \leq 360°)$ **[5]**

Question B5

a) A line l passes through point $A(-3,\ 2,\ 4)$ and point $B(9,\ -4,\ 22)$.

i. Write down a vector equation of line l. **[2]**

Point C lies at $(-3,\ 11,\ 21)$.

ii. Find the acute angle between AC and AB. **[4]**

iii. Show that triangle CAB is isosceles but not equilateral. **[3]**

iv Show that point $D(3, -1,\ 13)$ lies on AB. **[1]**

v. Find $\overrightarrow{AB}\,.\overrightarrow{CD}$ and explain what this shows. *Working must be shown*. **[3]**

b) A curve has equation $4x^2 - 2xy + y^2 = 48$.

i. Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

ii. Show that, where there is a stationary value, $y = 4x$. **[1]**

iii. Find the coordinates of the stationary values. **[2]**

Question B6

a) i. Use the substitution $u = 1 + x^3$ to find

$$\int \frac{x^2}{1 + x^3} \, dx.$$

[4]

ii. Solve the differential equation

$$\frac{dy}{dx} = \frac{yx^2}{1 + x^3}$$

subject to $y = 3$ when $x = 0$.

Write your answer in the form $y = f(x)$ which must contain no logarithms. **[4]**

b) The curve $y = \sqrt{(4 - x^2)}$ is rotated about the $x-$ axis between $x = 0$ and $x = 3$.

Find the volume formed. *All working must be shown*. **[4]**

c) Use integration by parts to evaluate

$$\int_0^{\pi} 4x \sin x \, dx.$$

[5]

d) A curve has equation

$$y = (x^2 - 8x + 3)^4 .$$

Find $\frac{dy}{dx}$ and state its value when $x = 4$. **[3]**

All working must be shown.

This is the end of the examination.

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