NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination 2017-18

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Point A lies at $(-6,\ 2)$ and point B lies at $(10,\ 6)$.

Find the equation of the line which passes through the mid-point of AB and is perpendicular to AB. **[4]**

Question A2

A box holds 4 blue beads and 5 green beads. One bead is taken and not replaced.

2 yellow beads are then added to the box and a second bead is taken.

Find the probability that **neither** bead is blue. **[3]**

Question A3

The quadratic equation $9x^2 + kx + 4 = 0$ has two real distinct roots.

Given that k is a positive integer, find the smallest value that it can take. **[3]**

Question A4

An arithmetic series has first term -50 and the 32nd term is 74.

a) Find the common difference. **[2]**

The sum of the first n terms is 0.

b) Find the value of n. **[3]**

Question A5

Solve the equation $9^{2x} - 4\,(9^x) + 3 = 0$. **[4]**

Question A6

Evaluate

$$\int_1^3 \left(1 - \frac{2}{x}\right)^2 \, dx.$$

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[4]**

Question A7

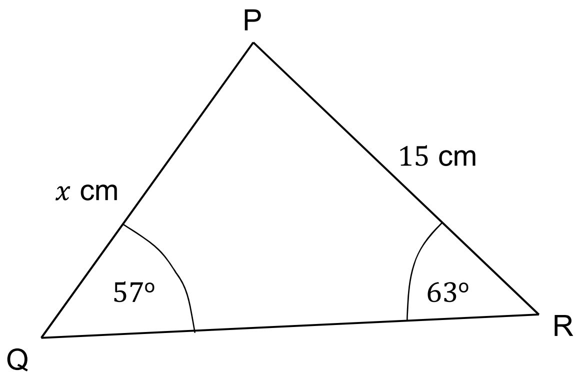

Figure 1

Figure 1 shows triangle PQR with PR = 15 cm, PQ = x cm, angle Q = 57° and angle R = 63°.

Find the value of x. Give your answer to **2** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[3]**

Question A8

The equation of a curve is given by $4x^3 - 2xy^2 + 12y = 5$.

Find $\frac{dy}{dx}$ in terms of x and y. *All working must be shown.* **[4]**

Question A9

Function $f(x)$ is defined as $f(x) = 3x + 1 \quad (-\infty < x < \infty)$

Function $g(x)$ is defined as $g(x) = x^2 - 1 \quad (-\infty < x < \infty)$

Solve the equation $f^{-1}(x) = g(x)$. *All working must be shown.* **[4]**

Question A10

The masses of bags of flour are assumed to follow a Normal distribution with standard deviation 7 grams.

A sample of 16 bags is selected and the mean mass is found to be 402.5 grams.

Find a 95% confidence interval for the masses of all the bags of flour. **[3]**

Question A11

Vector $\boldsymbol{a}$ is defined as $\boldsymbol{a} = 3p\boldsymbol{i} + p\boldsymbol{j} + 9\boldsymbol{k}$.

Find the values of p if $|\boldsymbol{a}| = 11$. **[3]**

Question A12

Use integration by parts to evaluate

$$\int_0^{\frac{\pi}{6}} 12x \cos x \;\, dx.$$

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[5]**

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a) Solve the inequality $3x^2 + x - 14 < 0$. **[4]**

b) i. By using the Factor Theorem, show that $(x - 5)$ is a factor of $2x^3 - 15x^2 + 13x + 60$. **[2]**

ii. Divide $2x^3 - 15x^2 + 13x + 60$ by $(x - 5)$. **[3]**

iii. Hence factorise $2x^3 - 15x^2 + 13x + 60$ completely. **[1]**

c) A geometric series has common ratio $\frac{5}{6}$ and the 5^{th} term is 1650 less than the 3^{rd} term.

i. Find the first term. **[3]**

ii. How many terms of the series are needed before its sum exceeds 40000? **[4]**

d) In the expansion of $(p + 2x)^7$ where $p \neq 0$, the coefficient of the term in x^4 is equal to $2240p^2$.

Find the value of p. **[3]**

Question B2

a) A large block of ice starts to melt and its mass, M kg, after t hours from when it starts to melt is given by the formula

$$M = 540\, e^{kt}$$

where k is a constant.

i. State the mass of the block when it starts to melt. **[1]**

After 2 hours the mass of the ice is 240 kg.

ii. Find the value of k. Give your answer in the form $\ln(\frac{m}{n})$ where m and n are integers. **[3]**

iii. Find the value of $\frac{dM}{dt}$ when $t = 5$. **[3]**

b) i. Solve the equation

$$3^{5x\,-\,2} = 729$$ **[2]**

ii. Solve the equation

$$2\log_8 x - \log_8(3x + 8) = \frac{1}{3} \;\; (x > 0)$$

<u>Each</u> stage of your working must be clearly shown. **[4]**

c)

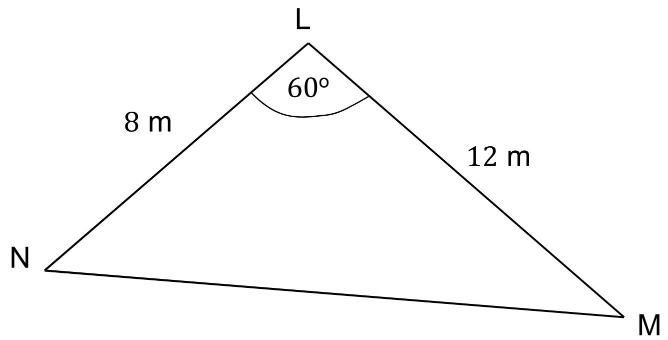

Figure 2

Figure 2 shows the acute-angled triangle LMN with LN $= 8$ m, LM $= 12$ m and angle L $= 60$°.

Find the length of NM. Give your answer in the form $4\sqrt{n}$ where n is an integer. **[3]**

d) Solve $\tan 2\theta = 4 \qquad (0° \le \theta \le 180°)$ **[4]**

Question B3

a) A curve has equation $y = \frac{1}{3}x^3 + x^2 - 3x + 2$.

i. Use $\frac{dy}{dx}$ to find the coordinates of the two points on the curve where there are stationary values. **[5]**

ii. Determine whether each stationary value is a maximum or a minimum. **[4]**

b)

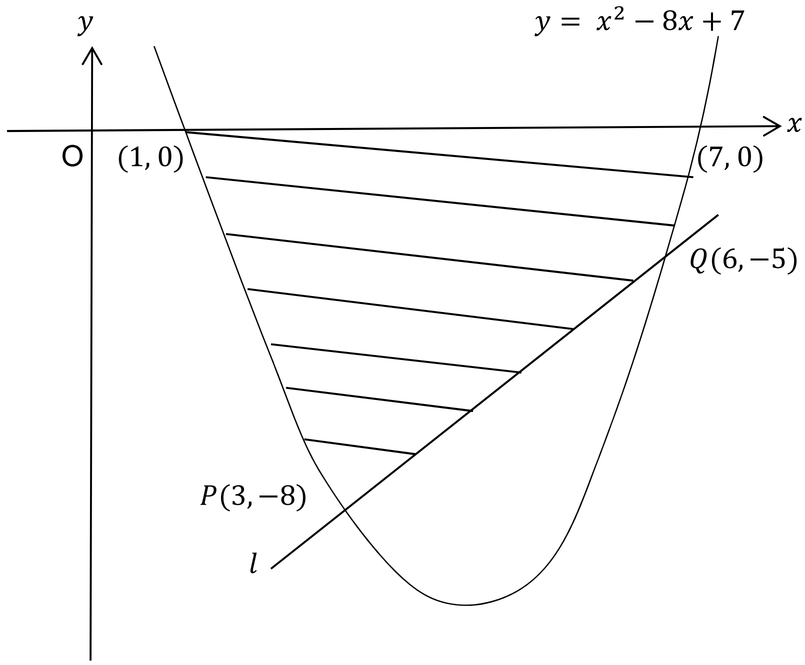

Figure 3

Figure 3 shows the curve $y = x^2 - 8x + 7$ and the line l. The curve $y = x^2 - 8x + 7$ crosses the $x-$ axis at points $(1, 0)$ and $(7, 0)$; and intersects line l at point $P(3, -8)$ and point $Q(6, -5)$.

i. Find the equation of the tangent to the curve at point P. **[3]**

ii. Hence confirm that line l is <u>not</u> a normal to the curve at point P. **[2]**

iii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = x^2 - 8x + 7$, line l and the $x -$ axis. **[6]**

Question B4

a)

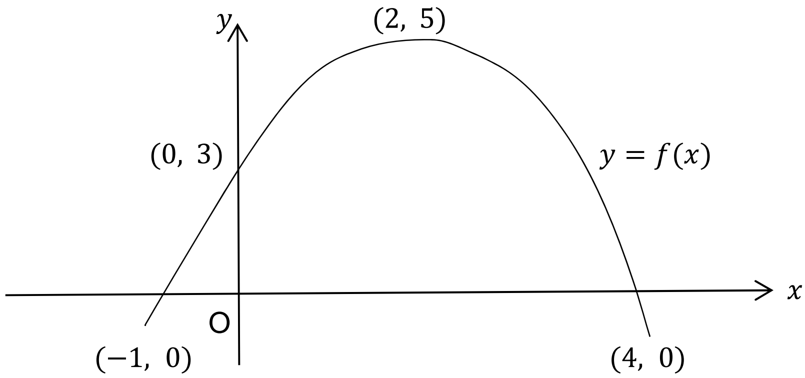

Figure 4

Figure 4 shows the graph of $y = f(x)$ which crosses the $x-$ axis at $(-1,\ 0)$ and $(4,\ 0)$; and crosses the $y-$ axis at $(0,\ 3)$. There is a stationary value at $(2,\ 5)$.

On two separate sets of axes, draw sketches of the following. On each sketch show clearly the coordinates of any stationary values, and where the curve crosses the $x-$ axis and the $y-$ axis.

i. $y = -f(x)$ **[3]**

ii. $y = f(x+2)$ **[3]**

b) Function $g(x)$ is defined as $g(x) = \frac{1}{x\ -\ 4}$ $(x > 4)$

Function $h(x)$ is defined as $h(x) = 3x + 5$ $(-\infty < x < \infty)$

i. State the range of $g(x)$. **[1]**

ii. Solve the equation $h(g(x)) = 9$. **[3]**

c) i. By using a suitable trigonometric formula, show that

$$\tan^2\theta = \sec^2\theta - 1.$$ **[2]**

ii. Solve the equation

$$\tan^2\theta - \sec\theta - 1 = 0 \quad (0 \le \theta \le 2\pi)$$ **[5]**

d) Angle A is acute and $\sin A = \frac{8}{17}$.

Without finding the size of angle A and showing all working, find $\sin 2A$. Give your answer in the form $(\frac{m}{n})$ where m and n are integers. **[3]**

Question B5

a) Line l_1 has equation $\boldsymbol{r} = (-\boldsymbol{i} + 6\boldsymbol{j} + 14\boldsymbol{k}) + \lambda(-2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$ where λ is a scalar.

Line l_2 has equation $\boldsymbol{r} = (-3\boldsymbol{i} + 22\boldsymbol{j} + 19\boldsymbol{k}) + \mu(-2\boldsymbol{i} + 6\boldsymbol{j} + 3\boldsymbol{k})$ where μ is a scalar.

i. Show that lines l_1 and l_2 intersect and find the coordinates of A which is their point of intersection. *All working must be shown*. **[6]**

The acute angle between lines l_1 and l_2 is θ.

ii. Write down an expression for $\cos\theta$ in the form $\frac{p}{q}$ where p and q are integers. **[3]**

iii. Show that point $B(9,\ 1,\ 4)$ lies on line l_1. **[1]**

You are given point $C(5, -2,\ 7)$ lies on line l_2.

iv. Find the lengths of AB, AC and BC giving your answers in surd form where appropriate, and show that

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos\theta.$$ **[3]**

b) A curve has equation

$$y = \frac{x^2 + 5}{x - 2}$$

Use the Quotient Rule to find $\frac{dy}{dx}$, and hence find the coordinates of the stationary values on the curve. **[5]**

c) Differentiate $\sin^5 x$. **[2]**

Question B6

a) The table below shows the values of $e^{\sqrt{x}}$ (given to two decimal places) for $x = 2, \quad 2.5, \quad 3, \quad 3.5$ and 4.

$\boldsymbol{x}$	2	2.5	3	3.5	4
$\boldsymbol{e^{\sqrt{x}}}$	4.11	p	5.65	6.49	7.39

i. Find the value of p. **[1]**

ii. Use the trapezium rule with 4 intervals to find an estimate of

$$\int_2^4 e^{\sqrt{x}}\ dx.$$ **[3]**

b) Solve the differential equation

$$\frac{dy}{dx} = y(2x - 1)$$

subject to $y = e$ when $x = 0$. Write your answer in the form $y = f(x)$.

All working must be shown. **[4]**

c) i. Write $\frac{3\ -\ x}{1\ -\ x^2}$ in the form $\frac{A}{1\ +\ x} + \frac{B}{1\ -\ x}$ where A and B are constants to be determined. **[3]**

ii. Hence evaluate

$$\int_{-\frac{1}{2}}^{0} \frac{3 - x}{1 - \ x^2}\ dx.$$

giving your answer in the form $\ln k$ where k is an integer.

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[5]**

d) Use the substitution $u = \ x^4 - 3$ to find

$$\int 48x^3(x^4 - 3)^5\ dx.$$ **[4]**

All working must be shown.

This is the end of the examination.

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