NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination

MARK SCHEME 2016-2017

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A7. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Uses any correct method to find one of the unknowns **[M1]**

Uses any correct method to find the second unknown **[M1]**

$c = -\frac{1}{2}$ or equivalent **[A1]**

$d = -3$ **[A1]**

Question A2

$\frac{7}{10} \times \frac{6}{9} \times \frac{3}{8}$ Any correct fraction, or its equivalent, seen **[M1]**

Multiplies their three fractions **[M1]**

$= \frac{7}{40}$, 0.175 or any equivalent answer. **[A1]**

Question A3

Discriminant $= 0.6^2 - 4 \times -0.1 \times -0.9$ (Allow this mark if either the 0.1 or the 0.9 has the wrong sign in front of it, but do not award the A mark in this case if the candidate then obtains 0) **[M1*]**

$= 0$ **[A1]**

There is one real root (allow also 'two equal roots'). (Allow follow through from their discriminant) **[A1ft]**

Question A4

$a \times 3^6 = 2916$ **[M1]**

$a = 4$ **[A1]**

$S_8 = \frac{\text{their } a\ (3^8 - 1)}{3 - 1}$ **[M1]**

$= 13120$ (allow 13100) **[A1]**

Question A5

Recognises the 'hidden quadratic equation' **[M1]**

Factorises or uses the formula
$[(4^x - 8)(4^x + 2) = 0 \text{ or } 4^x = \frac{6 \pm \sqrt{[(-6)^2 - 4 \times 1 \times -16]}}{2 \times 1}]$ **[M1]**

$4^x = 8 \ (\text{or} -2)$ (This mark is still awarded if the -2 is included) **[M1]**

$x = \frac{3}{2}$ or equivalent. (If any attempt is made to solve $4^x = -2$ this mark is lost) **[A1]**

Question A6

Attempts to differentiate (sight of $\frac{1}{x}$ or $\pm \frac{1}{x^2}$ is sufficient for this mark) **[M1*]**

$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$ **[A1]**

Sets their $\frac{dy}{dx}$ equal to 0 and finds a value for x. **[M1]**

Coordinates are $(1, 0)$ **[A1]**

Question A7

Angle B = 62° **[B1]**

$\frac{\text{AC}}{\sin 62} = \frac{5}{\sin 67}$ Correct use of the sine formula **[M1]**

$\text{AC} = 4.7959964$.... (metres) **[A1+]**

$= 4.80$ to 3 significant figures. **[A1ft]**

+This mark can be implied if 4.7959964.... is not seen but the 4.80 appears.
Allow follow through provided a more accurate answer is seen earlier.
<u>Special Case</u>: If the candidate finds BC $(= 4.2213...)$ and goes on to give it to 3 significant figures (4.22) then award B0 M1 A0 A1 ft.

Question A8

Median = 17.5 or equivalent **[B1]**

Multiplies their median by 8 and subtracts 143. **[M1]**

$x = -3$ **[A1]**

Question A9

Graph 1 – E **(B1)**; graph 2 – D **(B1)**; graph 3 – A **(B1)**; graph 4 – C **(B1)**. **[B4]**

Question A10

Confidence interval is 0.28 metres wide **[M1*]**

$\frac{1.96 \times 0.5}{\sqrt{n}} = 0.14$ **[M1*]**

$n = 49$ **[A1]**

Question A11

$50 \times 1.2^n = 1000 \times 0.8^n$ (**M1*** for LHS; **M1*** for RHS. Award even if the = sign is missing. If the 1.2^n and the 0.8^n are the wrong way round, award M1* only, but the next M1* and the following M1 can still be awarded.) **[M2*]**

[M1*]

Reaches an equation in index form $(1.5^n = 20)$

[M1]

Solves by any method.

[A1]

$(n \approx 7.4)$ shares reach equal value during 2016.

or After the M2*, the candidate can score the M1 mark by taking logs of both sides and applying the correct addition/subtraction law and the correct power law. The M1* is then awarded if the candidate reaches an expression of the form $n \log a = \log b$

Question A12

$\frac{du}{dx} = -\sin x \text{ or } du = -\sin x \, dx$ **[M1*]**

Writes integral in terms of u $(-\int_{\sqrt{3}/2}^{1/2} 2u^3 \, du)$. The limits do not have to be changed for this mark. **[M1*]**

Attempts to integrate (sight of u^4 is sufficient), substitutes the limits into their integrated expression and subtracts the right way round. If the limits have not been changed, then the integrated expression must be changed back into terms in x. **[M1]**

$= \frac{1}{4}$ or equivalent **[A1]**

If the answer appears with no working, this scores 0.

Section B

Question B1

a) i. Rearranges and reaches $x > \cdots$ **[M1]**

$x > \frac{16}{3}$ or equivalent **[A1]**

ii. Factorises or uses formula **[M1]**
$[(x-4)(x-9)=0$ or $x = \frac{13 \pm \sqrt{[(-13)^2 - 4 \times 1 \times 36]}}{2 \times 1}]$

Finds two critical values (4 and 9) **[M1]**

$x > 4$ **(A1)** and $x < 9$ **(A1)** or $4 < x < 9$ **(A1)** for each end **[A2]**

iii. 6, 7 and 8 **[B1]**

b) i. Sets $x = -2$ and substitutes in to $f(x)$. **[M1*]**

= - 91 **[A1]**

ii.

$$
\begin{array}{r l}
 & 3x - 7 \\
x^2 + 3 \,\big)\! & 3x^3 - 7x^2 + 9x - 21 \\
 & \underline{3x^3 \qquad\quad + 9x} \\
 & \qquad - 7x^2 \qquad - 21 \\
 & \qquad \underline{- 7x^2 \qquad - 21} \\
 & \qquad \underline{\quad \ldots\ldots\ldots\ldots \quad}
\end{array}
$$

One division correct **[M1]**

Second division correct **[M1]**

Correct quotient **[A1]**

c) i. $16 + (n-1)\times 3 = 58$ **[M1]**

$n = 15$ (or 15th day or 15th July) **[A1]**

ii. $S_{31} = \frac{31}{2}[2 \times 16 + (31-1)\times 3]$ $(= 1891)$ **[M1]**

Subtracts their answer from 2000 **[M1]**

109 left **[A1]**

d) Finds coordinates of points X and Y in terms of p $[(3p, 0)$ and $(0, p)]$. **[M1]**

$\frac{1}{2} \times$their $3p$ $\times$their $p = 54$ **[M1]**

$p = \pm 6$ (Both answers needed) **[A1]**

Question B2

a) i. $13 = 12\, e^{8k}$ and writes $e^{8k} = \cdots(\frac{13}{12})$ **[M1]**

Uses logs correctly ($8k = \ln(\frac{13}{12})$ **[M1]**

$k = \frac{1}{8}\ln(\frac{13}{12})$ or equivalent,or anything rounding to 0.01 **[A1]**

ii. *Please note: this is 'show that' question so all working must be seen.*

Substitutes $t = 40$ into their formula and obtains a value for P ($\approx 17.9\%$) **[M1*]**

Substitutes $t = 41$ into their formula and obtains a value for P ($\approx 18.1\%$) **[M1*]**

Thus concentration reached during 41^{st} day (or similar comment) **[A1]**

or Sets $P = 18$ and reaches a value for $e^{\text{their } k \times t}$ ($\frac{3}{2}$ or equivalent) **(M1*)**

Uses logs correctly and reaches a value for t **(M1*)**

$t =$ anything rounding to 40.5 followed by a comment. **(A1)**

b) $\log_2\left(\frac{x^2 + 3x}{x^2 - 9}\right) = \log_2 8^{2/3}$ Uses log subtraction law correctly **[M1*]**

Uses log power law correctly **[M1*]**

Removes logs at the correct time **[M1*]**

$\frac{x(x + 3)}{(x + 3)(x - 3)} = 4$ Factorises and cancels the $(x + 3)$, then rearranges to find a value for x. **[M1*]**

$x = 4$ **[A1]**

or If the factorising and cancelling does not take place, the fourth **M1*** is awarded for forming a quadratic equation $(3x^2 - 3x - 36 = 0)$ and attempting to solve it; the **A1** is given for $x = 4$ but is lost if the $x = -3$ solution is not discarded. (Placing $x = -3$ in brackets is good enough evidence of non-inclusion)

c) i. Uses cosine formula in any correct form ($189 = 12^2 + 15^2 - 2 \times 12 \times 15 \times \cos A$) **[M1]**

Rearranges correctly and finds a value for $\cos A$ (0.5) **[M1]**

$A = 60$ (degrees) [or $\frac{\pi}{3}$ or anything rounding to 1.05 (radians)] **[A1]**

ii. $\frac{1}{2} \times 12 \times 15 \times \sin$ their A **[M1]**

$= 45\sqrt{3}$ or equivalent or anything rounding to 78 (cm^2) **[A1ft]**

d) $\sin\theta = -\frac{7}{8}$ **[M1]**

$\theta =$ anything rounding to -61 (degrees) **[M1+]**

$\theta =$ anything rounding to 241 (degrees) **[A1]**

$\theta =$ anything rounding to 299 (degrees) **[A1]**

+The second M mark can be implied if the -61 is not seen but both answers are correct.

Question B3

a) i. $507\pi = 2\pi rh + \pi r^2$ **[M1]**

$h = \frac{507\pi - \pi r^2}{2\pi r}$ or $\frac{507}{2r} - \frac{r}{2}$ (or equivalent) **[A1]**

Please note: this is a 'show that' question so all working must be seen.

ii. Uses $V = \pi r^2 h$ **[M1*]**

Substitutes their value of h into their formula **[M1*]**

Reaches $V = \frac{507\pi r}{2} - \frac{\pi r^3}{2}$ with no errors seen **[A1]**

iii. $\frac{dV}{dr} = \frac{507\pi}{2} - \frac{3\pi r^2}{2}$ Attempts to differentiate (absence of r in first term or **[M1*]**

presence of r^2 in second term is sufficient evidence for this mark.

Correct answer **[A1]**

Sets their $\frac{dV}{dr}$ equal to 0 and finds a value for r^2 (169) **[M1]**

$r = 13$ (cm) **[A1]**

iv. $\frac{d^2V}{dr^2} = -\frac{6\pi r}{2}$ Attempts to differentiate a second time (presence of r is sufficient for this mark) **[M1*]**

Correct answer **[A1]**

This is negative, so there is a maximum (Reason and conclusion)
Allow follow through if the candidate obtains a negative $\frac{d^2V}{dr^2}$. **[A1ft]**

or Takes a numerical value below 13 and shows $\frac{dV}{dr} > 0$ **(M1*)**

Takes a numerical value above 13 and shows $\frac{dV}{dr} < 0$ **(M1*)**

Thus there is a maximum (or similar conclusion) **(A1ft)** Allow the follow through if the candidate's $\frac{dV}{dr}$ is positive for $r < 13$ and negative for $r > 13$.

b) i. Sets $e^x = 4 - e^x$, makes e^x the subject and reaches $x = \ln 2$ **[M1*]**

or substitutes $x = \ln 2$ into both equations shows the $y -$ value is the same in both cases.

$y = 2$ (The M mark does not need to have been scored, but the value must be in this form i.e. contains no logs or exponentials.) **[B1]**

ii. Divides area into two parts and indicates each area is

$\int_0^{\ln 2} e^x \, dx$ and $\int_{\ln 2}^{\ln 4} (4 - e^x) dx$ (The limits do not have to be correct at this stage) **[M1*]**

Attempts to integrate (presence of e^x or $4x$ is sufficient for this mark) **[M1]**

Both answers correct (e^x and $4x - e^x$)
[The correct limits for both areas must have been stated at some stage for this mark to be awarded] **[A1]**

Substitutes their limits into their integrated expressions and subtracts the right way round **[M1]**

Adds their areas **[M1]**

$= \ln 16 - 1$ or any equivalent exact form which does not have to be simplified. **[A1]**

Question B4

a) i.

Length of of call (t)	**Frequency (f)**	**Mid-value (x)**	$f \times x$	**Interval width**	**Frequency density**
$0 < t \leq 1$	7	0.5	3.5	1	7.0
$1 < t \leq 3$	17	2	34	2	8.5
$3 < t \leq 6$	21	4.5	94.5	3	7.0
$6 < t \leq 9$	15	7.5	112.5	3	5.0
$9 < t \leq 14$	10	11.5	115	5	2.0
	$\sum f = 70$		$\sum f \times x = 359.5$		

[M1]

Finds $\sum f \times x$ **[M1]**

Divides their $\sum f \times x$ by their $\sum f$ **[A1]**

Mean = anything rounding to 5.14

ii. $6 < t \leq 9$ (accept $6 - 9$) **[B1]**

iii. Correct frequency densities **[B1]**

(A sketch of the histogram is on page 14)

Correctly labelled and scaled horizontal axis **[B1]**

Boundaries of columns in correct places **[B1]**

Heights of columns correct to give appropriate areas. **[B1ft]**

Allow follow through for their frequency densities provided a reasonable histogram emerges.

(An accurate histogram which is not drawn on graph paper scores a maximum of 3 marks out of 4.)

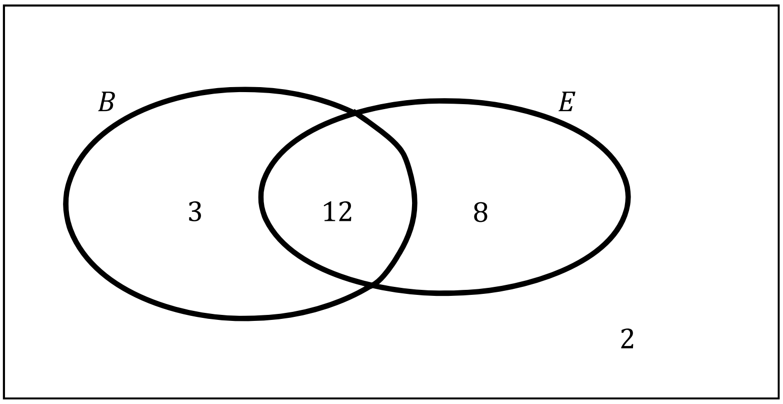
iv. $\frac{2}{3}$ of $21 + \frac{1}{3}$ of 15 or any indication, either in the script or on the graph, that a correct method is being used. **[M1]**

19 **[A1]**

Part b) is on the next page.

Question B4 – (continued)

b) i. E



(B1) for any one correct entry; **(B2)** for any three correct entries;
(B3) for all entries correct and presence of rectangle (but condone missing E). **[B3]**

ii. $p(B \cap E') = \frac{3}{25}$; $p(B' \cup E') = \frac{13}{25}$; $p(B \cup E)' = \frac{2}{25}$; $p(B|E) = \frac{12}{20}$.

(B1ft) for each one. Allow equivalent answers and follow through. **[B4ft]**

iii. States $p(B \cap E) \neq 0$ or that $p(B \cup E) \neq p(B) + p(E)$; or uses any valid wording like 'because they can happen at the same time' or 'one does not exclude the other'. **[B1]**

iv. *Please note: this is a 'show that' question so all working must be seen*.

Shows that $p(A) \times p(B) = p(A \cap B) = 0.48$ or equivalent

or shows $p(B|E) = p(B) = 0.6$ or equivalent

or shows $p(E|B) = p(E) = 0.8$ or equivalent **[M1*]**

Thus events B and E are independent (or similar conclusion) **[A1]**

Question B5

a) i. $\overline{x} = 12$ and $\overline{y} = 16$

$s_{xy} = \frac{863}{6} - 12 \times 16$ or anything rounding to -48.2 **[B1]**

$s_x^2 = \frac{1084}{6} - 12^2$ or anything rounding to 36.7 **[B1]**

$y - \text{their } \overline{y} = \frac{\text{their } s_{xy}}{\text{their } s_x^2} (x - \text{their } \overline{x})$ **[M1]**

$y = -1.3x + 32$ (Accept answers rounding to -1.3 and 32) **[A1]**

(If the equation appears with no working, award 1 mark out of 4.)

ii. Substitutes $x = 10$ into their equation **[M1]**

$y =$ anything rounding to 19 **[A1ft]**

b) Writes $\frac{a^2+19+37}{3} = 5a$ or shows an understanding of working out a moving average. **[M1]**

Forms a quadratic equation $(a^2 - 15a + 56 = 0)$ **[M1]**

Factorises or uses formula $[(a-7)(a-8) = 0 \text{ or } a = \frac{15 \pm \sqrt{[(-15)^2 - 4 \times 1 \times 56]}}{2 \times 1}]$ **[M1]**

$a = 7 \text{ or } 8$ (Both needed) **[A1]**

c) i. $z = \frac{100.4 - 96}{10} (= 0.44)$ **[M1]**

Finds Φ(their z) **[M1]**

= anything rounding to 0.67 **[A1]**

ii. Recognises Binomial application **[M1]**

${}^{36}C_{20}$ or ${}^{36}C_{16}$ × (their answer to i)^{20} ×(1 – their answer to i)^{16} **[M1]**

= anything rounding to 0.048 **[A1ft]**

d) i. $p = 0.2$ **[B1]**

$4.28 = 1 \times \text{their } p + 2 \times 0.24 + 6 \times 0.4 + q \times 0.16$ **[M1]**

$q = 7\frac{1}{2}$ or equivalent **[A1]**

ii. Any answer rounding to 2.5 **[B1]**

Question B6

a) Correct use of Chain Rule (sight of $6\cot^5 x$ or $-\csc^2 x$ is sufficient) **[M1]**

$-\csc^2 x \times 6\cot^5 x$ or equivalent **[A1]**

b) $12x - 3y - 3x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$ Use of Product Rule in its correct form **[M1*]**

Correct use of implicit differentiation (sight of $\pm 3x\frac{dy}{dx}$ or $\pm 6y^2\frac{dy}{dx}$ is sufficient) **[M1*]**

[M1*]

Factorises and obtains an expression for $\frac{dy}{dx}$ $\left(= \frac{3y - 12x}{6y^2 - 3x}\right)$ This mark can be given only if there are at least two terms in $\frac{dy}{dx}$. (If the candidate substitutes $x = 1$ and $y = -1$ before rearranging and goes on to obtain a numerical answer for $\frac{dy}{dx}$, then this mark can still be given).

Substitutes $x = 1$ and $y = -1$ into their $\frac{dy}{dx}$ **[M1]**

$= -5$ **[A1]**

c) Use of Quotient Rule in its correct form **[M1]**

$\frac{dy}{dx} = \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$ Correct answer (can be in unsimplified form) **[A1]**

Substitutes $x = 3$ into their $\frac{dy}{dx}$, inverts and changes sign (-2) **[M1]**

$y + 2 = -2(x - 3)$ or $y = -2x + 4$ or any other equivalent form. **[A1]**

d) i. $3x^2 - x - 1 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$ **[M1]**

$A = 1, \quad B = 2, \quad C = -1.$ **(A1)** for each **[A3]**

ii. Uses previous result and attempts to integrate (sight of $\ln$ ... in either of the first two terms or $\pm(x+1)^{-1}$ is sufficient for this mark) **[M1*]**

$\ln(x-2) + 2\ln(x+1) + \frac{1}{x+1}$ or equivalent **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Correct use of one of the log laws. **[M1]**

$= \ln 20 - \frac{1}{8}$ **[A1]**

Sketch of histogram for question B4 (not to scale)