NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination

2016-17

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**
The marks for each question are indicated in square brackets [ ].	

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Solve the simultaneous equations $4c - 3d = 7$

$$6c + 5d = -18$$ **[4]**

Question A2

A box holds 7 red beads and 3 green beads. Three beads are taken from the box, one after another with no replacement. Find the probability that the first two beads are red and the third bead is green. **[3]**

Question A3

A quadratic equation is defined as $-0.1x^2 + 0.6x - 0.9 = 0$.

Find the discriminant and hence state the number of real roots. **[3]**

All working must be shown.

Question A4

A geometric series has common ratio 3 and the 7^{th} term is 2916.

Find the first term, and the sum of the first 8 terms. **[4]**

Question A5

Solve the equation $4^{2x} - 6(4^x) = 16$. **[4]**

Question A6

A curve has equation $y = \ln x + \frac{1}{x} - 1 \quad (x > 0)$

Find the coordinates of its turning point. **[4]**

Question A7

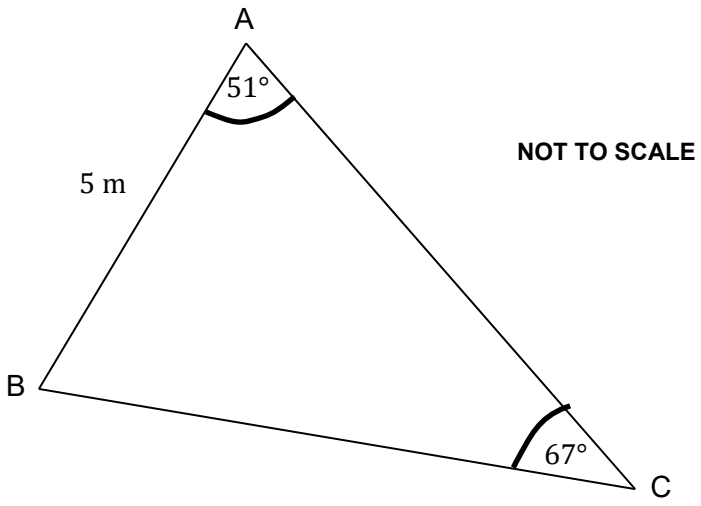


Figure 1

Figure 1 shows triangle ABC with AB = 5 metres, angle A = 51° and angle C = 67°.

Find the length of AC. Give your answer to **3** significant figures. **[4]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A8

Eight temperature readings (in ℃) are shown below in ascending (smallest to largest) order.

$$x, \quad 11, \quad 12, \quad 14, \quad 21, \quad 25, \quad 28, \quad 32.$$

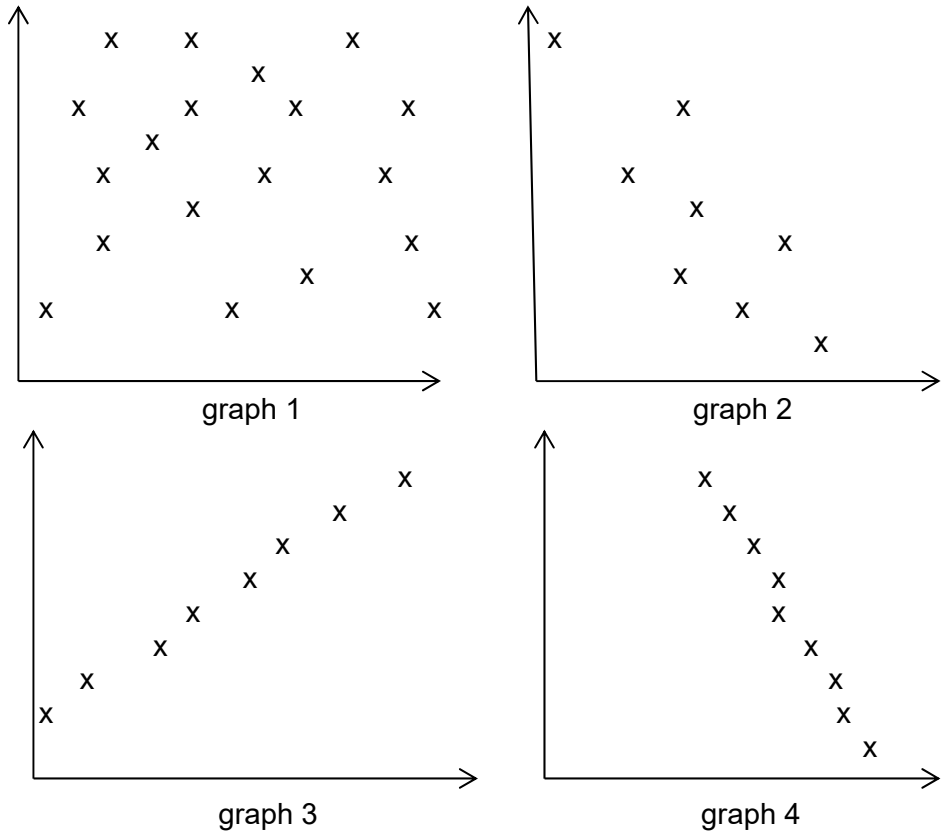
The mean and the median are the same.

Find the value of x. **[3]**

Question A9

Correlation can be described in 5 different ways:
A a strong positive correlation
B a weak positive correlation
C a strong negative correlation
D a weak negative correlation
E no correlation

Four scatter graphs are shown below. Match up each graph with one of the 5 descriptions above. (Note: each description can be used once, more than once or not at all) **[4]**



Question A10

Lengths of rope can be assumed to follow a Normal distribution. The standard deviation is 0.5 metres. A sample of n lengths gave a 95% confidence interval from 199.36 to 199.64 metres. Find the value of n. *Show all working*. **[3]**

Question A11

At the beginning of 2009, the price of a share in company X was £50 and the price of a share in company Y was £1000.

During each subsequent year, the price of the share in company X rose by 20% of its value and the price of the share in company Y fell by 20% of its value.

During which year did the two shares reach equal values? **[5]**

Question A12

Use the substitution $u = \cos x$ to evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2 \sin x \cos^3 x \ dx$$

[4]

Working must be shown. An answer on its own, even if correct, will score 0.

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) i. Solve the inequality $\frac{3x-1}{5} > 3$. **[2]**

ii. Solve the inequality $x^2 - 13x + 36 < 0$. **[4]**

iii. List the integers which satisfy **both** $\frac{3x-1}{5} > 3$ **and** $x^2 - 13x + 36 < 0$. **[1]**

b) The function $f(x)$ is defined as $f(x) = 3x^3 - 7x^2 + 9x - 21$.

i. Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(x + 2)$. **[2]**

ii. Divide $f(x)$ by $(x^2 + 3)$. *Show all working*. **[3]**

c) A man has 2000 ice creams in his freezer at the beginning of July in preparation for the summer season. On the first day, he sells 16 ice creams and on each subsequent day he sells 3 more ice creams than the previous day i.e. he sells 16 ice creams on 1st July, 19 on 2nd July, 22 on 3rd July, etc.

i. On which day does he sell 58 ice creams? **[2]**

ii. How many ice creams are left at the end of July (after 31 days)? **[3]**

d) The line $y = p - \frac{1}{3}x$ meets the $x-$ axis at point X and the $y-$ axis at point Y.

Point O is the origin.

The area of triangle OXY is 54 square units.

Find the possible values of p. **[3]**

Question B2

a) A farmer plants a field of sugar beet. When the beet reaches a certain size it is tested for sugar concentration. The first test showed a concentration of 12%.

It is believed that the percentage of sugar concentration, P, and the number of days after the first test. t, are connected by the formula

$$P = 12\, e^{kt}$$

where k is a constant.

i. After 8 days, the sugar concentration is 13%. Find the value of k. **[3]**

ii. When the sugar concentration reaches 18%, the sugar beet is ready to harvest. Show that this concentration is reached during the 41^{st} day. **[3]**

b) Solve the equation

$$\log_2(x^2 + 3x) - \log_2(x^2 - 9) = \frac{2}{3}\log_2 8 \qquad (x > 3).$$ **[5]**

Show all working.

c)

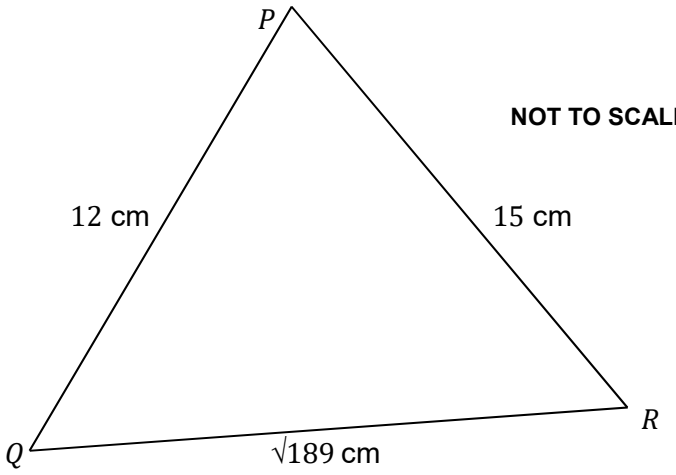


Figure 2

Figure 2 shows triangle PQR with $PQ = 12$ cm, $PR = 15$ cm and $QR = \sqrt{189}$ cm.

i. Find the size of angle P. **[3]**

ii. Find the area of triangle PQR. **[2]**

d) Solve the equation $8\sin\theta = -7$ for $0 \leq \theta \leq 360°$. **[4]**

Question B3

a)

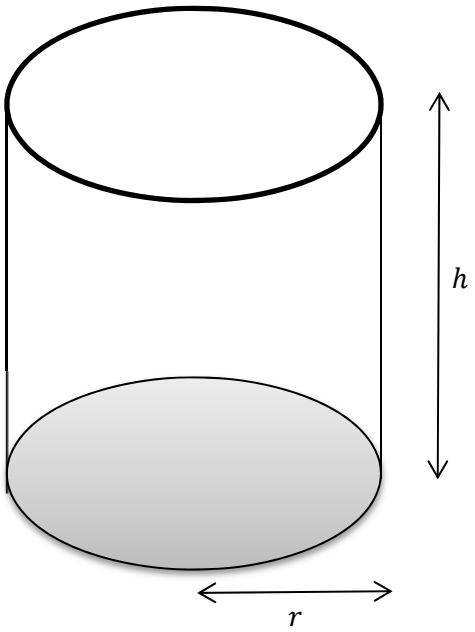


Figure 3

Figure 3 shows a hollow cylinder with radius r cm and height h cm.

The cylinder has a base ***but there is no top***.

The total outside surface area is 507π cm^2.

i. Find h in terms of r. **[2]**

ii. Show that the volume of the cylinder , V, is given by the formula

$$V = \frac{507\pi r}{2} - \frac{\pi r^3}{2}$$ **[3]**

iii. Use $\frac{dV}{dr}$ to find the value of r which gives the maximum volume. **[4]**

iv. Confirm that your volume is a maximum. **[3]**

Part b) is on the next page.

Question B3 – (continued)

b)

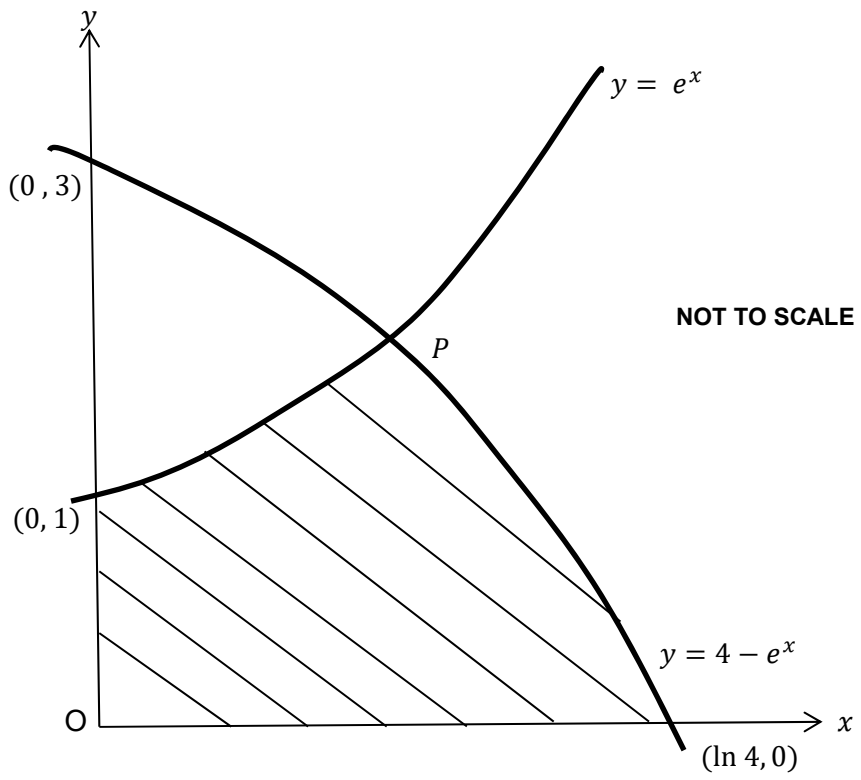


Figure 4

Figure 4 shows the curves $y = e^x$ and $y = 4 - e^x$. The curves meet at point P. The curve $y = e^x$ crosses the $y-$ axis at $(0, 1)$. The curve $y = 4 - e^x$ crosses the $x-$ axis at $(\ln 4, 0)$ and the $y-$ axis at $(0, 3)$.

i. Show that the $x-$ coordinate of point P is $\ln 2$ and find the $y-$ coordinate. **[2]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = e^x$, the curve $y = 4 - e^x$, the $x-$ axis and the $y-$ axis. **[6]**

Give your answer in **exact** form.

Question B4

a) During one day, 70 telephone calls were received by a large company. The lengths of the calls are summarised in the table below.

Length of call, t, in minutes	Frequency
$0 < t \leq 1$	7
$1 < t \leq 3$	17
$3 < t \leq 6$	21
$6 < t \leq 9$	15
$9 < t \leq 14$	10

(You may wish to copy and extend this table to help you answer some of the following questions.)

i. Estimate the mean. **[3]**

ii. In which interval does the upper quartile lie? **[1]**

iii. **On graph paper**, draw a histogram to show the data. **[4]**

iv. Using your histogram, or otherwise, estimate how many calls lasted between 4 and 7 minutes. **[2]**

b) In a group of 25 students, 15 study Biology, 20 study Economics and 2 study neither Biology nor Economics.

i. Draw a Venn diagram to show this information. **[3]**

One student is chosen at random. B denotes the event 'the student studies Biology' and E denotes the event 'the student studies Economics'.

ii. Find $p(B \cap E')$, $p(B' \cup E')$, $p(B \cup E)'$ and $p(B|E)$. **[4]**

iii. Explain why events B and E are **not** mutually exclusive. **[1]**

iv. Show clearly that events B and E **are** independent. **[2]**

Question B5

a) A coal merchant keeps a record of the mean temperature each month over the first 6 months of the year and the amount of coal sold in tonnes during that month. The results are recorded in the table below.

Mean temperature (x) in °C	**Amount of coal sold (y) in tonnes**
5	26
6	24
9	17
13	18
17	8
22	3

The data can be summarised as follows:

$$\sum x = 72; \quad \sum y = 96; \quad \sum x^2 = 1084; \quad \sum xy = 863.$$

i. Find ${s_x}^2$ and s_{xy}. Hence, find the equation of the regression line of y on x. Give your answer in the form $y = mx + c$. **[4]**

ii. Use your equation to predict the value of y when $x = 10$. **[2]**

b) The table below shows part of a record of car sales at a large garage over 3 months.

Month	**Number of cars sold**	**3 - point moving average**
1	a^2	
2	19	$5a$
3	37	

Find the two possible values of a. **[4]**

c) It is assumed that in a certain part of the world, the mean height of a 7-year old child is 96 cm with a standard deviation of 10 cm.

The heights of these children can be assumed to follow a Normal distribution.

i. What is the probability of a child being shorter than 100.4 cm? **[3]**

ii. In a group of 36 of these children, find the probability that exactly 20 children are shorter than 100.4 cm. **[3]**

d) i. A discrete random variable, X, has probability distribution as given in the table below:

x	1	2	6	q
$p(X = x)$	p	0.24	0.4	0.16

You are given $\mathrm{E}(X) = 4.28$. Find the value of p and the value of q. **[3]**

ii. You are also given $\mathrm{Var}(X) = 6.2416$. Find the standard deviation of X. **[1]**

Question B6

a) Differentiate $\cot^6 x$. **[2]**

b) A curve has equation $6x^2 - 3xy + 2y^3 = 7$.

Find the value of $\frac{dy}{dx}$ at the point $(1, -1)$. **[5]**

c) Find the equation of the normal to the curve

$$y = \frac{1+x}{1-x}$$

at the point $(3, -2)$. **[4]**

d) i. Write $\frac{3x^2 - x - 1}{(x-2)(x+1)^2}$ $(x \neq -1,\ 2)$ in the form $\frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

where A, B and C are constants to be determined. **[4]**

ii. Hence, evaluate

$$\int_3^7 \frac{3x^2 - x - 1}{(x-2)(x+1)^2}\ dx$$

giving your answer in the form $\ln p - \frac{1}{q}$ where p and q are integers. **[5]**

This is the end of the examination.