THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination

MARK SCHEME
2016-2017

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Finds the gradient of $3y - x - 8 = 0$ (= $\frac{1}{3}$) **[M1]**

Inverts their gradient and changes sign $(= -3)$ **[M1]**

Writes equation in correct form $y - 7 = \text{their gradient}(x + 3)$or equivalent **[M1]**

$3x + y + 2 = 0$ **[A1]**

Question A2

$0.3 \times x = 0.132$ **[M1]**

$x = 0.44$ **[A1]**

$y = 0.56$ (Follow through on their x) **[A1ft]**

Question A3

Factorises or uses the formula [$(2x - 3)(x + 5) = 0 \text{ or } x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times -15}}{2 \times 2}$] **[M1]**

Finds two critical values ($-5 \text{ and } \frac{3}{2}$) **[M1]**

$x \geq -5 \text{ and } x \leq \frac{3}{2}$ or equivalent **(A1)** for each **[A2]**
or $-5 \leq x \leq \frac{3}{2}$ **(A1)** for each end.

Question A4

(3 ×) $^8C_2 \times (\frac{1}{2})^6 \times (k(x))^2 = {}^8C_3 \times (\frac{1}{2})^5 \times (k(x))^3$
(Allow the presence of x and ^yC_x for ^xC_y. The 3 × … is not needed for this mark) **[M1*]**

Multiplies x^2 term by 3 or divides the x^3 term by 3 **[M1*]**

Reaches $k = \cdots$ (There must now be no x present) **[M1]**

$k = \frac{3}{4}$ **[A1]**

Question A5

$\log_9\left(\frac{x}{x-5}\right) = \log_9 3$ **(M1*)** for correct combining of logs on LHS **[M2*]**
(M1*) for writing RHS as a log

Cancels logs at the right time and solves to reach $x = \cdots$ **[M1]**

$x = \frac{15}{2}$ or equivalent (If the answer appears with no working, award 1 mark out of 4) **[A1]**

Question A6

$\text{Tan}\,\theta = -\frac{4}{5}$ **[M1]**

$\theta = 5.608444 \ldots$ **[A1+]**

$= 5.61$ to 3 significant figures. **[A1ft]**

(+This mark can be implied if 5.608444... does not appear but the 5.61 is seen. The follow through mark is available but a more accurate answer must be seen earlier.)

<u>Special case</u>: if a candidate works in degrees and writes 321.3 (without having shown a more accurate answer) give M1 A0 A1(ft).

Question A7

$f'(x) = 20x^4 + \frac{2}{x^2} + 3e^{3x} - \sec^2 x.$ (or any equivalent form) **[B3]**

(B1) any two correct; **(B2)** any three correct; **(B3)** all correct.

Question A8

$3.2 = \sqrt{(\frac{2137.35}{15} - m^2)}$

$10.24 = \frac{2137.35}{15} - m^2$ squares both sides **[M1]**

$m^2 = 132.25$ rearranges and reaches a value for m^2. **[M1]**

$m = 11.5$ or equivalent **[A1]**

Question A9

$p = 530$ **[B1]**

$q = 194$ **[B1]**

Any sensible comment *e.g.* only small amount of data available; readings over too short a time interval; the summer of 2017 could be quite different from the summer of 2016 which could affect sales; etc. **[B1]**

Question A10

$E(X) = (1 \times 0.16) + (2 \times 0.3) + (3.5 \times 0.24) + (6 \times 0.12) + (8 \times 0.18)$ **[M1]**

$= 3.76$ **[A1]**

$E(X^2) = (1 \times 0.16) + (4 \times 0.3) + (12.25 \times 0.24) + (36 \times 0.12) + (64 \times 0.18)$ **[M1]**

$(= 20.14)$

$\text{Var}(X) = \text{their } E(X^2) - [\text{their } E(X)]^2$ **[M1]**

= anything rounding to 6 **[A1]**

Question A11

$-8x + 2y^3 + 6xy^2\frac{dy}{dx} + 16\frac{dy}{dx} = 0$ Correct use of Product Rule. **[M1*]**

Correct attempt at implicit differentiation either $6xy^2\frac{dy}{dx}$ or $16\frac{dy}{dx}$ is sufficient evidence to award this mark. **[M1*]**

Factorises and reaches an expression in $\frac{dy}{dx}$ $\left(\frac{8x - 2y^3}{6xy^2 + 16}\right)$. (This mark is only available if there is more than one $\frac{dy}{dx}$ term in the expression. If the candidate substitutes $x = 2$ and $y = -2$ before correctly rearranging and goes on to obtain a numerical answer for $\frac{dy}{dx}$, then this mark can still be given). **[M1*]**

Substitutes $x = 2 \text{ and } y = -2 \text{ into their } \frac{dy}{dx}$ **[M1]**

$= \frac{1}{2}$ or equivalent. **[A1]**

Question A12

$\frac{du}{dx} = 3x^2$ or $du = 3x^2\, dx$ **[M1*]**

Writes integral in terms of u. ($\int_1^2 \frac{1}{3}\, e^{2u}\, du$) (The limits do not need to have been changed for this mark) **[M1*]**

Attempts to integrate, substitutes limits into their integrated expression and subtracts the right way round. If the limits have not been changed, then the expression must be turned back into terms in x before this mark can be given. **[M1]**

$= \frac{1}{6}(e^4 - e^2)$ or any equivalent form but it must be exact. **[A1]**

Section B

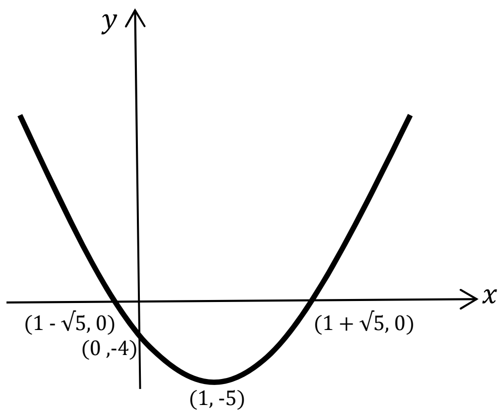
Question B1

a) i. $(x-1)^2 - 1 - 4 = 0$ or $(x-1)^2 - 5 = 0$ **[M1*]**

Reaches $(x-1) = \pm\sqrt{\dots}$ **[M1*]**

$x = 1 + \sqrt{5}$ or $1 - \sqrt{5}$ **(A1)** for each or **(A2)** for $1 \pm \sqrt{5}$ **[A2]**

ii.



(B1) Correct shape drawn anywhere

(B1) Minimum at (1,-5)

(B1) (0, -4), (1 - √5, 0) and (1 + √5, 0) shown
Allow decimal equivalents here - anything rounding to (-1.24, 0) and (3.24, 0) **[B3]**

b)

$$\begin{array}{r|l} & 2x^2 - 11x + 12 \\ x+2 & 2x^3 - 7x^2 - 10x + 24 \\ & 2x^3 + 4x^2 \\ & \quad -11x^2 - 10x \\ & \quad -11x^2 - 22x \\ & \qquad 12x + 24 \\ & \qquad 12x + 24 \\ & \qquad \dots\dots\dots \end{array}$$

Attempts first division **[M1]**

Attempts any correct subsequent division **[M1]**

Correct quotient and no errors **[A1]**

$f(x) = (x+2)(2x-3)(x-4)$ **[A1]**

c) $a + (n-1)d = -100$ using their a and their d and obtains a value for n. **[M1]**

($n = 80\frac{7}{8}$) so the 81st term is the first to fall below -100. **[A1]**

d) i. Writes $\frac{ar^6}{ar^3} = \frac{1822.5}{67.5}$ or the other way up **[M1]**

Rearranges and reaches $r^3 = \cdots$ or $\frac{1}{r^3} = \cdots$ (27 or $\frac{1}{27}$) **[M1]**

$r = 3$ **[A1]**

Substitutes r into either formula and obtains a value for a. **[M1]**

$a = 2.5$ or equivalent. **[A1]**

ii. $S_{12} = \frac{\text{their a(their r}^{12} - 1)}{\text{their r} - 1}$ or equivalent **[M1]**

$= 664300$ (Accept 664000) **[A1]**

Question B2

a) i. $£16$ **[B1]**

ii. *Please note: (1) this is a 'show that' question so all working must be seen; (2) a candidate who works backwards scores 0 marks.*
Uses $20 = 16e^{4k}$ and reaches an expression in e^{4k} $(\frac{20}{16})$ **[M1*]**

Uses logs correctly **[M1*]**

$k = \frac{1}{4}\text{In}\frac{5}{4}$ (this must be seen) $= 0.0558$ to 3 significant figures. **[A1]**

iii. $P = 16\, e^{8 \times 0.0558}$ **[M1]**

= anything rounding to 25 (pounds) **[A1]**

iv. $\frac{dP}{dt} = 16 \times k \times e^{kt}$ **[M1]**

Substitutes $t = 5$ into their $\frac{dP}{dt}$ **[M1]**

= anything rounding to 1.18 (pounds per week) **[A1]**

v. 'After 5 weeks the rate of price increase is £1.18 per week' or similar words ['5 weeks' must be mentioned and 'rate per week' must also be stated or implied (just seeing 'per week' is sufficient)] Allow follow through as this mark is for interpretation – part iv does not have to be correct. **[B1ft]**

b) $\log_2 125^{2/3} + \log_2(\sqrt{12})^4 - \log_2 15^2$ (Correct use of power law) **[M1*]**

$= \log_2 25 + \log_2 144 - \log_2 225$ (Correct simplification) **[M1*]**

$= \log_2 16$ (Correct use of addition/subtraction laws) **[M1*]**

$= 4$ (If the answer appears with no working, this scores 0) **[A1]**

c) i. Uses the Cosine Formula in any form $\left(\cos\theta = \frac{20^2 - 16^2 - 18^2}{-2 \times 16 \times 18}\right)$ **[M1]**

Correct manipulation **[M1]**

$= \frac{5}{16}$ **[A1]**

ii. Uses the formula, a right-angled triangle or any valid method **[M1*]**

$= \frac{\sqrt{231}}{16}$ (M mark scored and no errors seen) **[A1]**

iii. $9\sqrt{231}$ (m^2) **[B1]**

Question B3

a) i. $48\pi = \frac{1}{2} \times \pi \times 2r \times l + \pi r^2 (2 \times \frac{1}{2})$ **[M1]**

Reaches $l = \frac{48\pi - \pi r^2}{\pi r}$ or $\frac{48}{r} - r$ or any equivalent form **[A1]**

Please note: this is a 'show that' question so all working must be seen.

ii. Uses $V = \frac{1}{2}\pi r^2 l$ **[M1*]**

Substitutes their l into their expression for V $\left[V = \frac{1}{2}\pi r^2\left(\frac{48}{r} - r\right)\right]$ **[M1*]**

$V = 24\pi r - \frac{1}{2}\pi r^3$ (Both M marks scored and no errors seen) **[A1]**

iii. Attempts to differentiate (No r in first term or sight of r^2 in second term is sufficient evidence for this mark) **[M1*]**

$\frac{dV}{dr} = 24\pi - \frac{3}{2}\pi r^2$ **[A1]**

Sets their $\frac{dV}{dr}$ equal to 0 and reaches $r^2 = \cdots (16)$ **[M1]**

$r = 4$ **[A1]**

iv. Attempts to differentiate a second time (Presence of r is sufficient for this mark) **[M1*]**

$\frac{d^2V}{dr^2} = -3\pi r$ **[A1]**

This is negative, so there is a maximum (Reason and conclusion) **[A1ft]**

(Follow through can be given provided their $\frac{d^2V}{dr^2}$ is negative for their value of r.)

or Takes a numerical value below 4 and shows that $\frac{dV}{dr} > 0$ **(M1*)**

Takes a numerical value above 4 and shows that $\frac{dV}{dr} < 0$ **(M1*)**

Thus there is a maximum (conclusion needed) **(A1ft)**

(Allow follow through if there is a maximum for their value of r and their value of $\frac{dV}{dr}$.)

b) i. Substitutes $x = 6$ into both equations and shows that $y = 24$

or sets equations equal to each other, solves, and finds $x = 6$ followed by a substitution into one equation to find the $y -$ value. **[A1]**

Sufficient working *must* be shown to earn the A mark.

ii. Area of triangular section $= \frac{1}{2} \times 6 \times 24$ or integrates $4x$ between 0 and 6 and reaches as far as substituting the limits into their integrated expression and subtracting the right way round. **[M1]**

$= 72$ **[A1]**

Recognises the need to integrate $16x - 2x^2$ between 6 and 8 **[M1]**

Attempts to integrate (Presence of x^2 in the first term or x^3 in the second term is sufficient for this mark) ($= 8x^2 - \frac{2}{3}x^3$) **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

Adds their areas **[M1]**

$= \frac{296}{3}$ or equivalent, or anything rounding to 98.7 **[A1]**

If the correct answer appears with little or no working, M1 A1 M1 M0 M1 M1 A0 is possible.

Question B4

a)

Time, t, in min	Frequency (f)	Mid-value (x)	$f \times x$	Cum. fr.
$4 \leq t \leq 5$	6	4.5	27	6
$5 < t \leq 6$	12	5.5	66	18
$6 < t \leq 7$	23	6.5	149.5	41
$7 < t \leq 8$	22	7.5	165	63
$8 < t \leq 9$	12	8.5	102	75
$9 < t \leq 10$	8	9.5	76	83
$10 < t \leq 11$	5	10.5	52.5	88

i. Adds up $f \times x$ column (638) **[M1]**

Their $\sum f \times x \div$ their $\sum f$ $(638 \div 88)$ **[M1]**

$= 7.25$ or equivalent **[A1]**

ii. Finds cumulative frequencies **[M1]**

Plots correct curve (a sketch is on page 16). 1 mark lost for each omitted/incorrect plot; 1 mark lost for each point missed by the curve by at least 1 mm (but allow ft for any incorrect plots); 1 mark lost if either axis not labelled correctly. **[A3]**

(<u>Please note</u>: a maximum total of 3 marks can be lost i.e. there are no negative scores. If the candidate plots the mid-values instead of the upper values in each interval, this will score A0.)

If graph paper is not used, award 1 mark out of the A3 if a reasonable curve is drawn. If a cumulative frequency polygon is drawn, award up to 2 marks out of the A3.

iii. Finds correct median from their curve (about 7.1 for a correct curve) **[B1ft]**

Finds lower quartile and upper quartile from their curve (about 6.2 and 8.2 respectively for a correct curve) **[B1ft]**

Subtracts their lower quartile from their upper quartile (about 2 for a correct curve) **[B1ft]**

vi. Correct reading taken from their curve (about 18 minutes if the curve is correct) **[B1ft]**

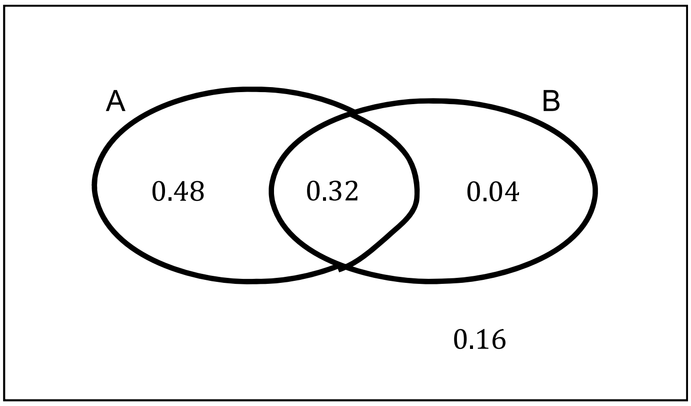
b) i. Uses $p(B|A) = \frac{p(B \cap A)}{p(A)}$ **[M1*]**

$0.4 = \frac{0.32}{p(A)}$ giving $p(A) = \frac{4}{5}$ or 0.8 (M mark scored and no errors seen) **[A1]**

Uses $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ **[M1*]**

$p(B) = 0.36$ or equivalent **[A1]**

ii. E



A and B overlapping enclosed in a rectangle (Condone missing E). **[M1]**

0.32 and 0.48 in correct places **[A1]**

0.04 and 0.16 in correct places [follow through from their $p(B)$] **[A1ft]**

$p(A' \cup B) = 0.52$ **[A1]**

iii. Verifies that $p(A \cap B) \neq p(A) \times p(B)$ or $p(A|B) \neq p(A)$ or $p(B|A) \neq p(B)$ and states a conclusion. **[A1]**

Question B5

a) i. $s_x = \sqrt{[156.5 \div 8 - 4^2]}$ (≈ 1.89) **[B1]**

$s_y = \sqrt{[25590 \div 8 - 56^2]}$ (≈ 7.92) **[B1]**

$s_{xy} = 1799 \div 8 - 4 \times 56$ (≈ 0.875) **[B1]**

$r =$ anything rounding to 0.06 **[B1]**

ii. Almost no association shown or words to this effect (allow 'very (or more extreme) weak positive correlation') **[B1]**

Claim cannot be supported (follow through on their correlation coefficient) **[B1]ft**

b) i. $\Phi(z) = 0.8$ leading to a value of z (0.84) **[M1]**

$\frac{52 - x}{10} = \text{their } z$ or equivalent **[M1]**

$x =$ anything rounding to 43.6 (grams) **[A1]**

ii. $(z =)\ \frac{67.3 - 52}{10}$ or equivalent $(= 1.53)$ **[M1]**

Finds Φ(their z) (0.937) and subtracts from 1 **[M1]**

$= 6.3\%$ (must be a percentage) **[A1]**

iii. Recognises Binomial $(30,$ their $0.063)$ **[M1]**

$p(X = 3) = {}^{30}C_3 \times$ their $0.063^3 \times$ their 0.937^{27} **[M1]**

= anything rounding to 0.175 **[A1]**

c) i. $7225 \div 0.85$ or equivalent **[M1]**

$= £8500$ **[A1]**

ii. Either $7225 \times 0.85^n = 2500$ or their $8500 \times 0.85^n = 2500$ **[M1]**

Rearranges, takes logs correctly and reaches $n = \cdots$ (either $6.53 \ldots$ or $7.53 \ldots$) **[M1]**

(During) 2023 **[A1]**

Question B6

a) i. Correct use of the Chain Rule (sight of $\sec^2 x$ or $8\tan^7 x$ is sufficient for this mark) **[M1]**

$8\tan^7 x \sec^2 x$ **[A1]**

ii. Correct use of Quotient Rule **[M1]**

$(\frac{dy}{dx} =)\ \frac{(x - 3) - x}{(x - 3)^2}$ **[A1]**

Sets their $\frac{dy}{dx}$ equal to $-\frac{4}{3}$ and attempts to solve by any valid method. **[M1]**

Finds at least one value of x and substitutes into original equation to find a value of y **[M1]**

Coordinates are $(4.5, 3)$ and $(1.5, -1)$ or equivalent (both needed) **[A1]**

(If the correct answers appear with no working, award 1 mark out of 5.)

b) i. $12 = A(x + 3) + B(x - 3)$ **[M1]**

$A = 2$ and $B = -2$ **(A1)** for each **[A2]**

ii. Attempts to integrate (presence of a log term is sufficient for this mark) **[M1]**

$2\ln(x-3) - 2\ln(x+3)$ **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Uses any logarithmic law (power, addition or subtraction) **[M1]**

$= \ln 16$ (must be in this form) **[A1]**

c)

Uses integration by parts in the right direction **[M1*]**

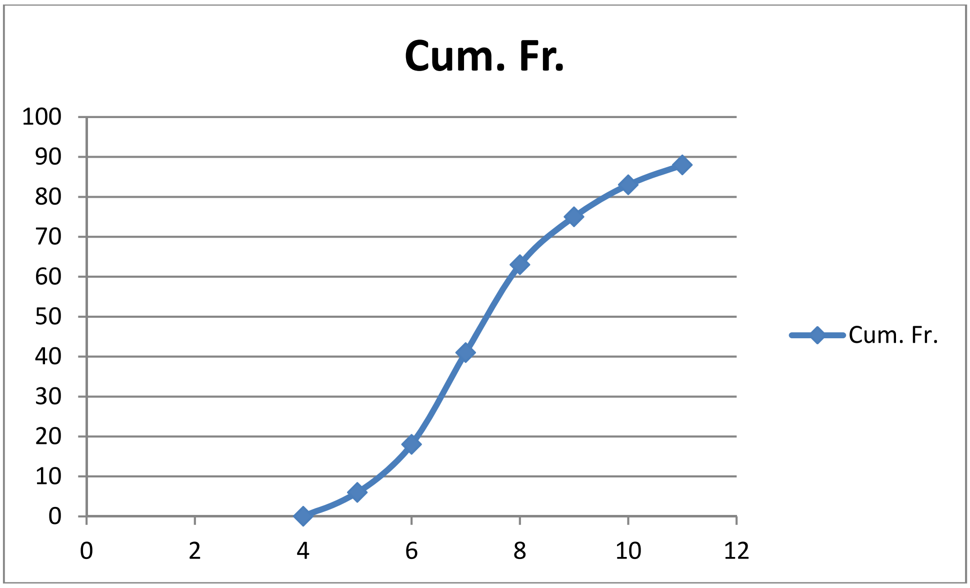
$= 16x \times \frac{1}{4}e^{4x}$ **(A1)** - $\int_0^{1/4} 16 \times \frac{1}{4}e^{4x}\,dx$ [A1 for correct first part] **[A1]**

$= 4x\,e^{4x} - e^{4x}$ or equivalent (Correct integrated expression) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 1$ (If the answer appears with no working, this scores 0 marks) **[A1]**

Cumulative Frequency curve for question B4



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