NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business (Business)

2016-17

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**
The marks for each question are indicated in square brackets [ ].	

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

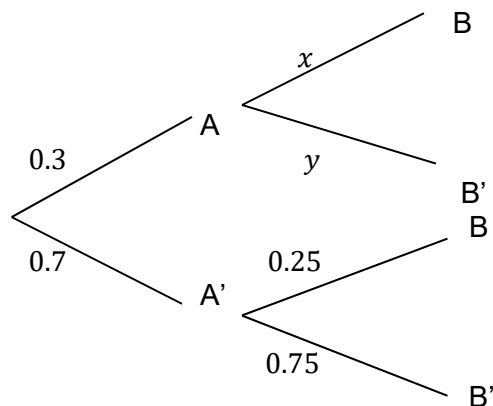**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR**

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Find the equation of the line which is perpendicular to the line $3y - x - 8 = 0$ and passes through the point $(-3, \ 7)$.

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.　　　**[4]**

Question A2

The tree diagram above shows the probabilities of events A and B.　The probability that event A and event B both occur is 0.132

Find the value of x and the value of y.　　　**[3]**

Question A3

Find the range of values which satisfy the inequality $2x^2 + 7x - 15 \leq 0$.　　　**[4]**

Question A4

In the expansion of $(\frac{1}{2} + kx)^8$, where $k \neq 0$, the coefficient of the x^3 term is 3 times larger than the coefficient of the x^2 term.

Find the value of k.　　*Show all of your working.*　　　**[4]**

Question A5

Solve the equation $\log_9 x - \log_9(x-5) = \frac{1}{2}$ $(x > 5)$. *Show all of your working.* **[4]**

Question A6

Solve the equation $5\tan\theta = -4$ for $\pi \le \theta \le 2\pi$.

Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[3]**

Question A7

The function $f(x)$ is defined as $f(x) = 4x^5 - \frac{2}{x} + e^{3x} - \tan x$.

Find $f'(x)$. **[3]**

Question A8

A set of 15 readings has standard deviation 3.2 and the sum of the squares of the readings is 2137.35

Find the mean. **[3]**

Question A9

A small shop sells ice-creams. The numbers of ice-creams sold during each quarter of 2016 and during the first quarter of 2017 are shown in the following table.

Year	Quarter	Number of ice-creams sold	4-point moving average
2016	1	170	
	2	588	
			p
	3	1043	
			536
	4	319	
2017	1	q	

Find the value of p and the value of q.

Give a reason why it may not be appropriate to use the readings above to predict ice-cream sales during the rest of 2017. **[3]**

Question A10

A discrete random variable, X, has probability distribution as given in the table below.

x	1	2	3.5	6	8
$p(X = x)$	0.16	0.3	0.24	0.12	0.18

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[5]**

Question A11

A curve has equation $-4x^2 + 2xy^3 + 16y = -80$.

Find the value of $\frac{dy}{dx}$ at the point $(2, -2)$. *All working must be shown.* **[5]**

Question A12

Use the substitution $u = x^3 + 1$ to find the **exact** value of

$$\int_0^1 x^2\, e^{2(x^3+1)}\ dx$$ **[4]**

All working must be shown.

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a) i. Solve the equation $x^2 - 2x - 4 = 0$ **by completing the square**.

   You <u>must</u> show each stage of your working.

   Give your answers in the form $a + \sqrt{b}$ and $a - \sqrt{b}$ where a and b are integers. **[4]**

   ii. Sketch the graph of $y = x^2 - 2x - 4$. Show clearly the coordinates where your curve has a stationary value and where it crosses the $x -$ and $y -$ axes. **[3]**

b) The function $f(x)$ is defined as $f(x) = 2x^3 - 7x^2 - 10x + 24$.

   Divide $f(x)$ by $(x + 2)$ and hence factorise $f(x)$ completely. *Working must be shown.* **[4]**

c) An arithmetic progression is defined as 539, 531, 523, ….

   Which term in the progression is the first to fall below -100? **[2]**

d) The 4th term of a geometric series is 67.5 and the 7th term is 1822.5

   i. Find the common ratio and the first term. **[5]**

   ii. Find the sum of the first 12 terms. **[2]**

Question B2

a)   Shares in a company become available and the price of a single share, £P, is given by the formula

$$P = 16e^{kt}$$

where t is the number of weeks from when the shares first become available and k is a constant.

i.   State the value of a share when $t = 0$.   **[1]**

After 4 weeks the price of a share has risen to £20.

ii.   Show that the value of k is approximately 0.0558   **[3]**

Each stage of your working must be clearly shown.

iii.   Find the price of a share after 8 weeks.   **[2]**

iv.   Find the value of $\frac{dP}{dt}$ when $t = 5$.   **[3]**

v.   Explain what your answer to part iv tells us.   **[1]**

b)   Write the expression $\frac{2}{3}\log_2 125 + 4\log_2(\sqrt{12}) - 2\log_2(15)$ in its simplest form which must contain no logarithms.   **[4]**

Each stage of your working must be clearly shown.

Part c) is on the next page.

Question B2 – (continued)

c)

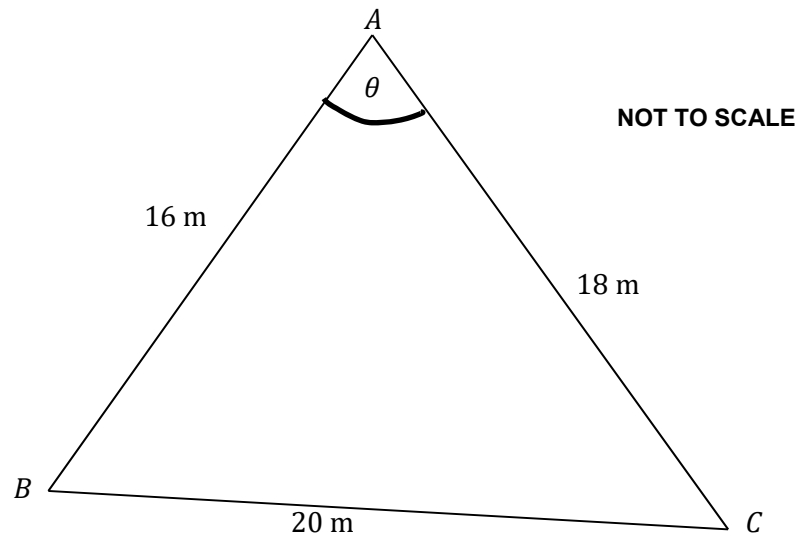

A

θ

NOT TO SCALE

16 m

18 m

B

20 m

C

Figure 1

Figure 1 shows the acute-angled triangle ABC with

$AB = 16$ m, $AC = 18$ m, $BC = 20$ m and angle $A = \theta$.

i. Find the value of $\cos \theta$, giving your answer in the form $\frac{a}{b}$ where a and b are integers. **[3]**

ii. *Without* finding the size of θ, and showing *all* working, show that the exact value of $\sin \theta$ is $\frac{\sqrt{231}}{16}$. **[2]**

iii. Find the area of triangle ABC giving your answer in the form $m\sqrt{n}$ where m and n are integers. **[1]**
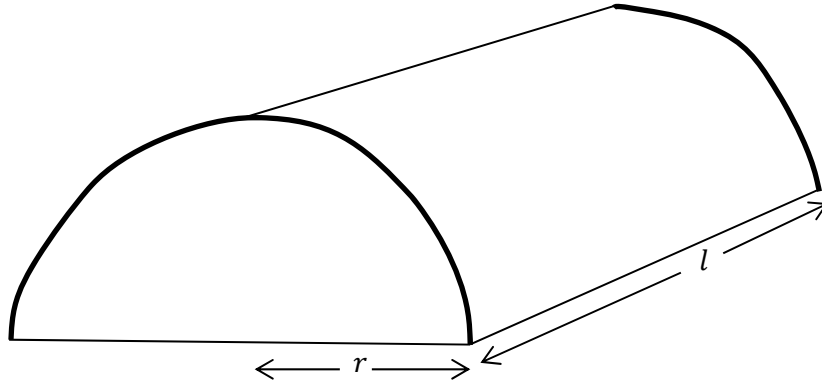
Question B3

a)

Figure 2

Figure 2 shows a large shed which has length l metres and cross-section in the shape of a semi-circle with radius r metres.

The shed has no base.

The outside surface area of the shed is $48\pi \text{ m}^2$.

i.   Find l in terms of r.   **[2]**

ii. Show that the volume, V, of the shed is given by

$$V = 24\pi r - \frac{1}{2}\pi r^3.$$ **[3]**

iii. Use $\frac{dV}{dr}$ to find the value of r which gives the maximum volume.   **[4]**

iv. Confirm that your volume is a maximum.   **[3]**

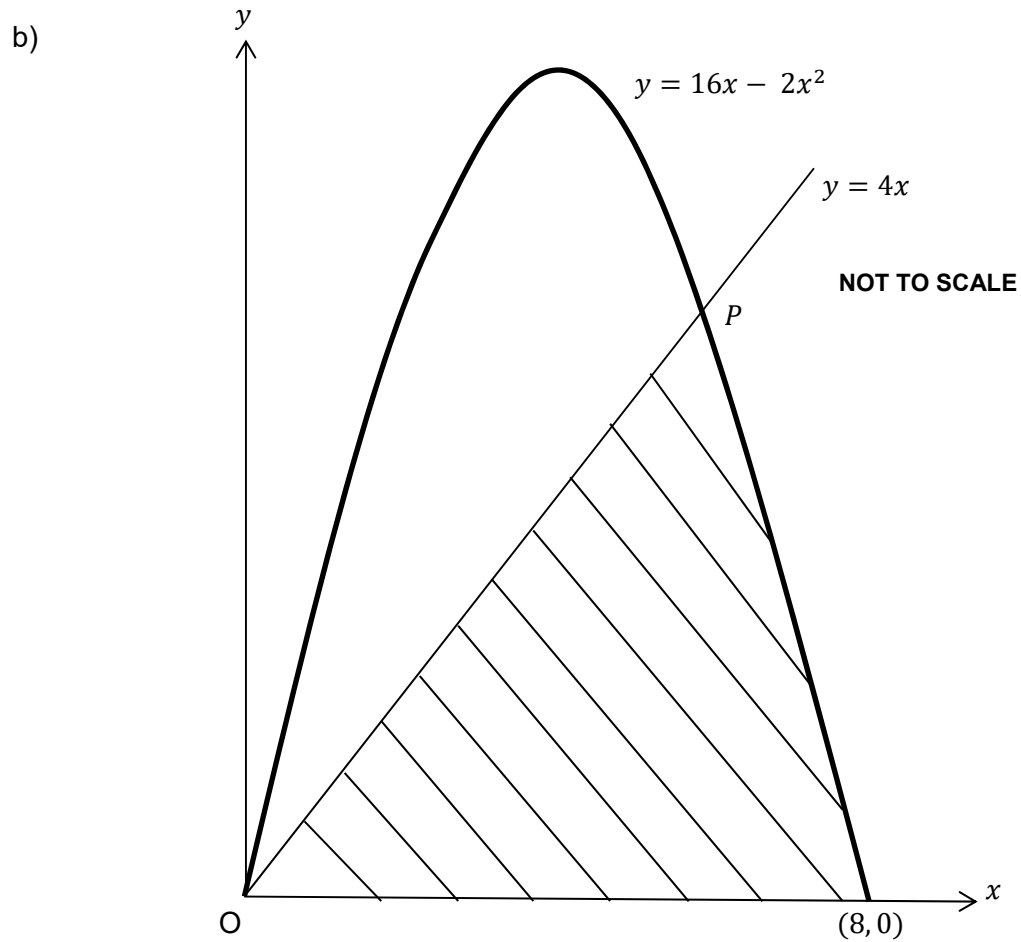**Part b) is on the next page.**

Question B3 – (continued)

b)

Figure 3

Figure 3 shows the curve $y = 16x - 2x^2$ and the line $y = 4x$. The curve and the line intersect at the origin and at point P.

i. Confirm that the coordinates of point P are $(6, 24)$. **[1]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = 16x - 2x^2$, the line $y = 4x$ and the $x -$ axis. **[7]**

Show all working.

Question B4

a) An assistant at a large department store kept a record of the number of minutes spent serving each customer over a period of time. The results are shown in the table below.

Time, t, in minutes	Frequency
$4 \le t \le 5$	6
$5 < t \le 6$	12
$6 < t \le 7$	23
$7 < t \le 8$	22
$8 < t \le 9$	12
$9 < t \le 10$	8
$10 < t \le 11$	5

(You may wish to copy and extend this table on your answer booklet to help you answer some of the following questions.)

i. Estimate the mean. **[3]**

ii. **On graph paper**, draw a cumulative frequency curve. **[4]**

iii. Use your cumulative frequency curve to estimate the median and interquartile range. **[3]**

iv. Use your cumulative frequency curve to estimate how many customers took longer than $8\frac{1}{2}$ minutes to serve. **[1]**

b) Two events, A and B, are such that $p(B|A) = 0.4$, $p(A \cap B) = 0.32$ and $p(A \cup B) = 0.84$.

i. Show that $p(A) = 0.8$ and find $p(B)$. **[4]**

ii. Draw a Venn diagram to show these probabilities and hence work out $p(A' \cup B)$. **[4]**

iii. Show that events A and B are **not** independent. **[1]**

Question B5

a) It is claimed that a certain substance, S, when added to cattle food will result in an increase in milk production. A farmer decides to try out this substance. She adds a fixed amount of substance S to the cattle food each month and also records the milk production. The trial lasts 8 months. The results are shown in the table below.

Amount of substance S added (kg) (x)	Milk production (in hundreds of litres to the nearest whole number) (y)
1	50
6	70
3.5	52
4.5	64
5	48
2	57
3	61
7	46

The data can be summarised as follows:

$$\sum x = 32; \sum y = 448; \sum x^2 = 156.5; \sum y^2 = 25590; \sum xy = 1799.$$

i.   Find s_x, s_y and s_{xy}. **[4]**

Hence find the product moment correlation coefficient.

ii. Describe the correlation between the amount of substance S added and the milk production. Can the claim be supported? **[2]**

b) The masses of hens' eggs can be assumed to follow a Normal distribution with mean 52 grams and standard deviation 10 grams.

i.   20% of the eggs have a mass of x grams or less. Find the value of x. **[3]**

ii. If an egg has a mass of 67.3 grams or more, it is classified as 'very large'. What percentage of all eggs are 'very large'? **[3]**

iii. A tray holds 30 eggs. Find the probability that there are exactly 3 'very large' eggs. **[3]**

c) A car depreciates by 15% of its value each year. At the beginning of 2017 the car is worth £7225.

i.   Find its value at the beginning of 2016. **[2]**

ii. In which year will the value of the car be expected to fall below £2500? **[3]**

Question B6

a)   i.   Differentiate $y = \tan^8 x$.   **[2]**

     ii.   A curve has equation $y = \dfrac{x}{x-3}$.

         There are two points on the curve where the gradient is $-\dfrac{4}{3}$.

         Find the coordinates of these two points. *Working must be shown.*   **[5]**

b)   i.   Express $\dfrac{12}{x^2-9}$ in the form $\dfrac{A}{x-3} + \dfrac{B}{x+3}$ where A and B are constants

         to be determined.   **[3]**

     ii.   Hence evaluate

$$\int_{4}^{11} \frac{12}{x^2 - 9} \; dx$$

         giving your answer in the form $\ln k$ where k is an integer.   **[5]**

c)   Use integration by parts to evaluate

$$\int_{0}^{\frac{1}{4}} 16x \, e^{4x} \; dx$$

     [5]

All working must be shown: just quoting the answer, even if it is correct, will score no marks.

This is the end of the examination.