NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination
2017-18

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**
The marks for each question are indicated in square brackets [ ].	

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

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Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Point A lies at $(-6, -2)$ and point B lies at $(10, -6)$.

Find the equation of the line which passes through the mid-point of AB and is perpendicular to AB. **[4]**

Question A2

A box holds 4 blue balls and 6 yellow balls. One ball is removed and not replaced.

Three green balls are then added to the box. Another ball is removed.

Find the probability that the two balls removed from the box are both blue. **[4]**

Question A3

Divide $2x^3 + x^2 - 13x + 6$ by $(x - 2)$. **[3]**

Question A4

The 8th term of an arithmetic series is 15 times larger than the first term.

a) Show that, if a is the first term and d the common difference, then $d = 2a$. **[2]**

The sum of the first 20 terms is 1600.

b) Find a and d. **[3]**

Question A5

Solve the equation $6^x = 1850$

Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[3]**

Question A6

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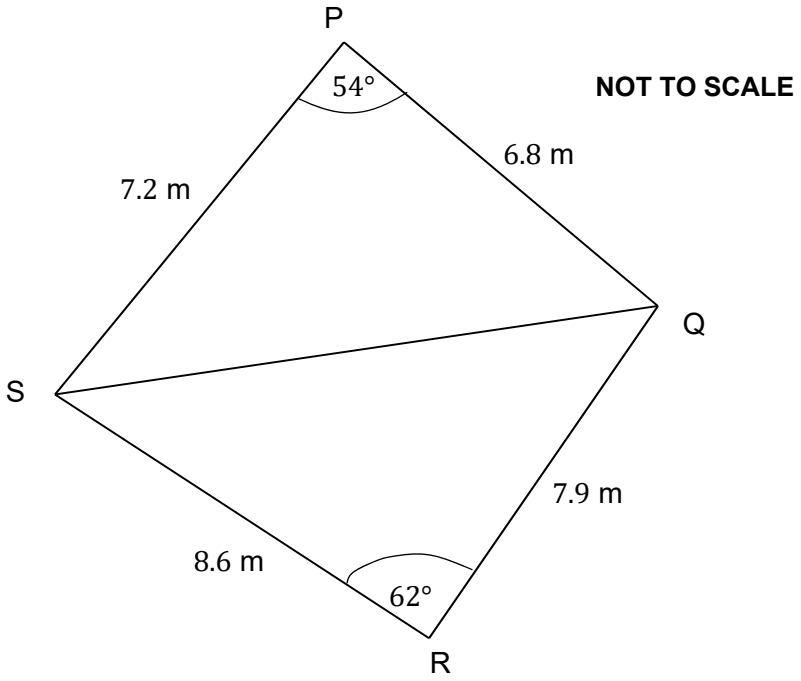

Figure 1

Figure 1 shows a quadrilateral PQRS which is made up of two acute-angled triangles PQS and SQR. PS $= 7.2$ m, PQ $= 6.8$ m, SR $= 8.6$ m, QR $= 7.9$ m, angle P $= 54^{o}$ and angle R $= 62^{o}$.

Find the area of the quadrilateral PQRS. **[4]**

Question A7

Differentiate $\frac{3}{x} + \cos x - e^{-3x}$ **[3]**

Question A8

Evaluate

$$\int_{3}^{6} \left(\frac{1}{x} - 2\right) dx$$

Give your answer in the form $\ln a - b$ where a and b are integers.

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[3]**

Question A9

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For events A and B, $p(A) = \frac{1}{4}$, $p(B) = \frac{3}{5}$ and $p(A \cap B) = \frac{1}{8}$.

Find $p(A \cup B)$ and $p(B|A)$. **[4]**

Question A10

A student invests £2000 for 5 years.

The interest rate is 5% per year for the first 3 years. It then drops to 4% per year for the remaining 2 years.

Find the total compound interest earned over the 5 years. **[4]**

Question A11

A large box contains marbles. 70% of the marbles are red and the remaining 30% are yellow.

20 marbles are selected at random.

Find the probability that exactly 13 or exactly 14 marbles are red. **[4]**

Question A12

Use the substitution $u = x^4 - 2$ to evaluate

$$\int_1^0 \frac{x^3}{(x^4-2)^3}\ dx.$$

Give your answer in the form $\frac{m}{n}$ where m and n are integers.

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[4]**

Section B

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Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) i. Solve the inequality $3(4x - 3) < 8x + 11$ **[2]**

ii. Solve the inequality $x^2 + 5x - 14 \geq 0$ **[4]**

iii. List the positive integers which satisfy **both** $3(4x - 3) < 8x + 11$ **and** $x^2 + 5x - 14 \geq 0$ **[1]**

b) When $x^2 + 7x - 5$ is divided by $(x + k)$ the remainder is $-3k$.

Use the Remainder Theorem to find the possible values of k. **[4]**

c) In the expansion of $(p - 2x)^7$, where $p \neq 0$, the coefficient of the x^3 term is the same as the coefficient of the x^4 term.

Find the value of p. **[3]**

d) The second term of a geometric series is 1536 and the 7th term is $364\frac{1}{2}$.

i. Find the common ratio and the first term. **[5]**

ii. Investigate whether the sum of the series will ever reach 8200. **[1]**

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Question B2

a) A certain type of bird was introduced on to an island and the number of birds, N, after t years from when they were first introduced is given by the formula

$$N = 75e^{0.2t} + 25$$

i. How many birds were introduced on to the island? **[1]**

ii. Find the number of birds after 6 years. **[2]**

iii. After how many years were there 192 birds? **[3]**

iv. Find the value of $\frac{dN}{dt}$ when $t = 10$. **[3]**

v. Explain what your answer to part iv tells us. **[1]**

b)

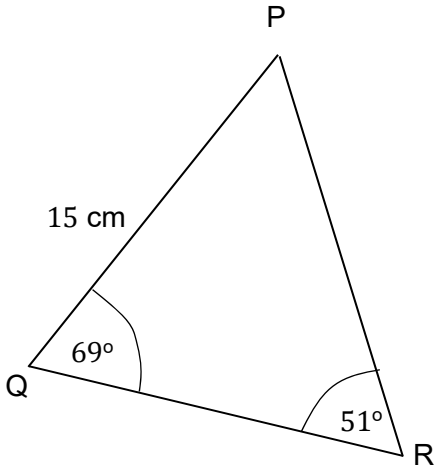

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Figure 2

Figure 2 shows acute-angled triangle PQR with PQ $= 15$ cm, angle Q $= 69°$ and angle R $= 51°$.

Find the perimeter of triangle PQR. **[5]**

c) Solve the equation $\cos 2\theta = -\frac{3}{8} \quad (0° \leq \theta \leq 360°)$. **[5]**

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Question B3

a)

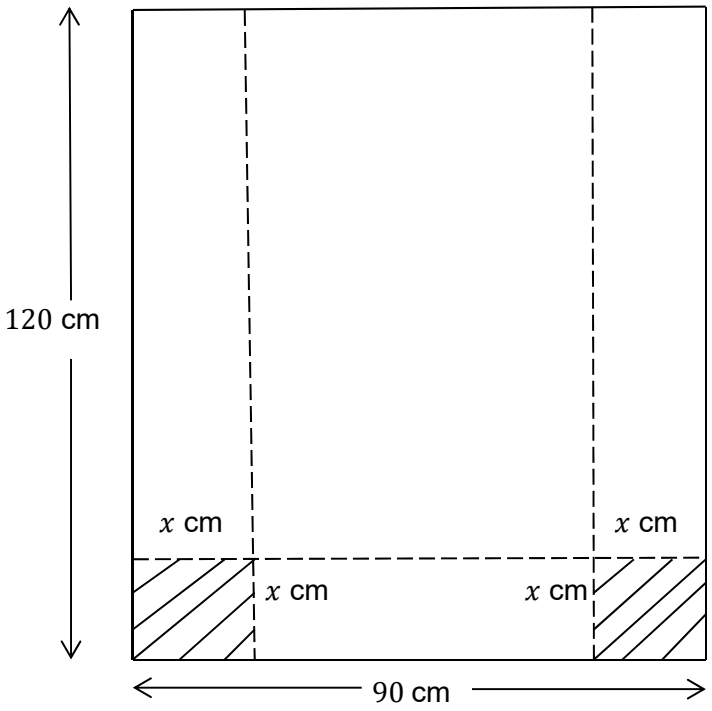

Figure 3

Figure 3 shows a rectangular sheet of metal which is 120 cm long and 90 cm wide. Two squares, x cm by x cm are cut from two of the corners, and the remaining three sides are folded up along the dotted lines to make a tray which has one end missing.

i. Show that the volume of the tray, V, is given by

$$V = 2x^3 - 330x^2 + 10800x.$$ **[2]**

ii. Use $\frac{dV}{dx}$ to find the value of x which gives the maximum volume. **[4]**

iii. Confirm that your volume is a maximum. **[3]**

iv. State the maximum volume. **[1]**

Part b) is on the next page.

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Question B3 – (continued)

b)

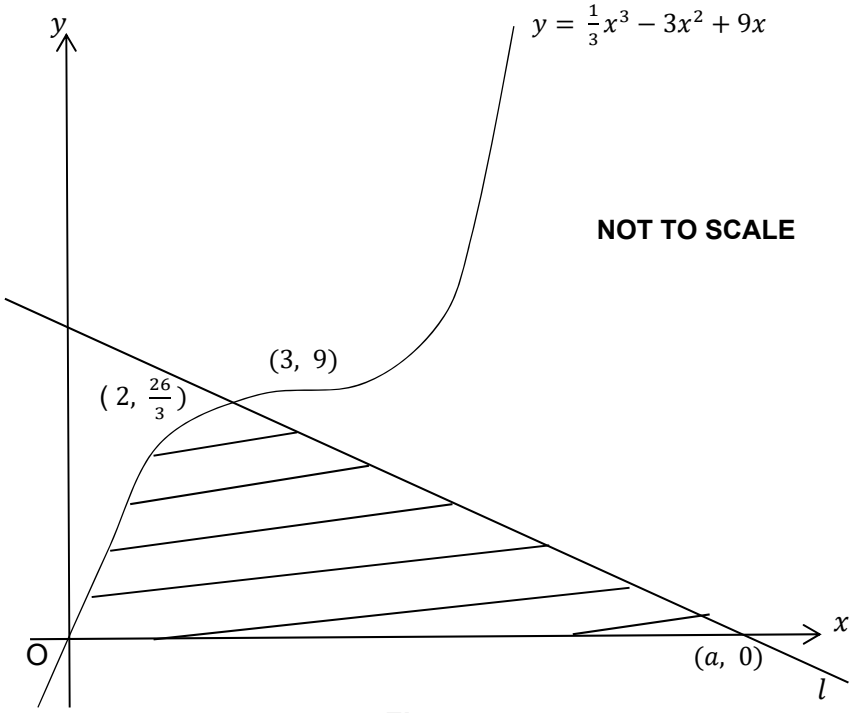

Figure 4

Figure 4 shows the curve $y = \frac{1}{3}x^3 - 3x^2 + 9x$ and the line l which is a normal to the curve at the point $\left(2, \frac{26}{3}\right)$.

There is a stationary value on the curve at the point $(3, 9)$.

i. What type of stationary value is at the point $(3, 9)$? **[1]**

ii. Find the equation of line l.

Give your answer in the form $y = mx + c$. **[3]**

Line l meets the $x-$ axis at $(a, 0)$.

iii. State the value of a. **[1]**

iv. Find the area, which is shaded on the diagram, that is bounded by the curve $y = \frac{1}{3}x^3 - 3x^2 + 9x$, line l and the $x-$ axis **[5]**

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Question B4

a) During one week, 50 trains were late arriving at a station. The numbers of minutes, t, that each train was late are summarised in the table below.

Number of minutes late (t)	Frequency
$0 < t \leq 2$	14
$2 < t \leq 4$	10
$4 < t \leq 8$	16
$8 < t \leq 12$	6
$12 < t \leq 20$	4

(You may wish to copy and extend this table to help you answer some of the questions below.)

i. Estimate the mean. **[3]**

ii. In which interval does the lower quartile lie? **[1]**

iii. On graph paper, draw a histogram to show the data. **[4]**

iv. Using your histogram, or otherwise, estimate how many trains were between 3 and 5 minutes late. **[2]**

b) An archer fires two arrows at a target. The probability that the first arrow hits the target is 0.7. If he hits the target with his first shot, the probability that he hits the target with his second is 0.8. If he misses the target with his first shot, the probability that he hits the target with his second is 0.6.

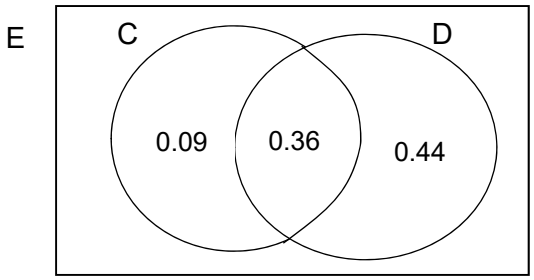
i. Draw a fully labelled tree diagram. **[2]**

ii. Find the probability that the archer hits the target with his second shot. **[3]**

You are given that the archer hits the target with his second shot.

iii. Find the probability that he missed with his first shot. **[2]**

c)



The Venn diagram shows the probabilities concerning events C and D.

State, with a reason if events C and D are:

i. mutually exclusive **[1]**

ii. independent. **[2]**

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Question B5

a) The numbers of hot drinks sold at a roadside café during one hour, and the outside temperature, were recorded over 5 days in February. The results are shown in the table below.

Temperature (x)	**Number of drinks sold** (y)	x^2	xy
0	10		
-2	11		
3	4		
5	2		
4	3		

i. Copy and complete the table. **[2]**

ii. Find s_x^2 and s_{xy} , and hence find the equation of the regression line of y on x. *Show all working.* **[4]**

iii. Use your equation to estimate the number of drinks sold when the temperature is -4℃. **[2]**

iv. How reliable is your estimate in part iii? Give a reason. **[1]**

b) The masses of bags of corn can be assumed to follow a Normal distribution with standard deviation 0.4 kg.

A sample of 49 bags is selected and the mean mass is found to be 19.8 kg.

i. Find a 95% confidence interval for the mean mass of corn contained in all of the bags. **[3]**

The label on each bag claims that the mass of the contents is 20 kg.

ii. Comment on this claim. **[1]**

c) A discrete random variable, X, has probability density as given in the table below.

x	0	1	4	6	q
$p(X = x)$	p	0.25	0.35	0.18	0.12

i. Find the value of p. **[1]**

ii. You are given $\mathrm{E}(X) = 3.93$. Find the value of q. **[2]**

iii. Find $\mathrm{Var}(X)$. **[3]**

iv. Another discrete random variable, Y, is defined as $Y = 7 - 4X$.

Write down $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. **[1]**

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Question B6

a) A curve, C, has equation $3x^2 + 3xy - y^2 = -28$.

i. Find an expression for $\frac{dy}{dx}$ in terms of x and y. *Show all working*. **[4]**

Curve C has two stationary values.

ii. At each stationary value, show that $y = -2x$. **[1]**

iii. Find the coordinates of both stationary values. **[3]**

b) By writing $\tan x$ as $\frac{\sin x}{\cos x}$, and using the Quotient Rule, show that the differentiation of $\tan x$ is $\frac{1}{\cos^2 x}$. **[3]**

c) i. Express $\frac{3x^2 + 10x + 9}{(x+1)(x+2)^2}$ in the form $\frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ where A, B and C are constants to be determined. **[4]**

ii. Hence evaluate

$$\int_0^1 \frac{3x^2 + 10x + 9}{(x+1)(x+2)^2}\ dx.$$

Give your answer in the form $\ln a - \frac{1}{b}$ where a and b are integers.

All working must be shown. An answer, even the correct one, will receive no marks if this working is not seen. **[5]**

This is the end of the examination.

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