

# NCUK

## THE NCUK INTERNATIONAL FOUNDATION YEAR

### IFYMB002 Mathematics Business Examination 2017-18

**Examination Session**  
Semester Two

**Time Allowed**  
2 Hours 40 minutes  
(including 10 minutes reading time)

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### INSTRUCTIONS TO STUDENTS

**SECTION A** Answer ALL questions. This section carries 45 marks.

**SECTION B** Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

**DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE  
INVIGILATOR**

## Section A

**Answer ALL questions. This section carries 45 marks.**

**Question A1**

Line  $l$  passes through point  $A(-20, k)$ , point  $B(-8, 2)$  and point  $C(6, 9)$ .

Find the value of  $k$ . **[ 4 ]**

**Question A2**

A student plays two games of badminton. The probability that she wins the first game is 0.65. If she wins the first game, the probability that she wins the second is 0.8. If she does not win the first game, the probability that she wins the second is 0.4.

a) Draw a fully labelled tree diagram. **[ 2 ]**

b) Find the probability that she wins exactly one game. **[ 3 ]**

**Question A3**

Find the range of values which satisfy the inequality  $2x^2 - 3x - 20 \leq 0$ . **[ 4 ]**

**Question A4**

Find the two possible values of  $p$  if the coefficient of the term in  $x^4$  in the expansion of  $(p - 3x)^6$  is 135. **[ 3 ]**

**Question A5**

Solve the equation

$$\log_8(x - 4) + \log_8(x - 5) = \frac{1}{3} \quad (x > 5).$$

**[ 5 ]**

*All working must be shown.*

**Question A6**

Solve the equation  $2 \cos \theta = -1$  ( $0 \leq \theta \leq 2\pi$ ).

Give your answers as exact multiples of  $\pi$ . **[ 3 ]**

**Question A7**

An equation is defined as  $y = x^3 + \ln x$ .

Find the value of  $\frac{d^2y}{dx^2}$  when  $x = 0.7$ , giving your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[ 4 ]**

**Question A8**

The heights in cm ( $x$ ) and the masses in kg ( $y$ ) of some children were recorded. The product moment correlation coefficient was 0.8 and the equation of the line of regression of  $y$  on  $x$  was  $y = 0.4x + 5$ .

It is discovered afterwards that the machine used for measuring the masses was faulty, giving readings that were all 0.2 kg above the true readings.

Write down the new product moment correlation coefficient and the new equation of the line of regression. **[ 3 ]**

**Question A9**

Two events,  $A$  and  $B$ , are such that  $p(B|A) = 0.75$ ,  $p(A \cap B) = 0.6$  and  $p(A \cup B) = 0.55$ .

Find  $p(A)$  and  $p(B)$ . **[ 4 ]**

**Question A10**

The masses of bags of rice are assumed to follow a Normal distribution with mean 700 grams and standard deviation 20 grams.

What percentage of bags will have a mass of more than 735 grams? **[ 3 ]**

**Question A11**

A curve has equation

$$y = \frac{1 + x^3}{x^2 - 1}.$$

Find the equation of the tangent at the point  $(3, \frac{7}{2})$ . **[ 4 ]**

**Question A12**

Use integration by parts to find

$$\int 8x \cos 2x \, dx. \quad \text{[ 3 ]}$$

**Section B**  
**Answer 4 questions ONLY. This section carries 80 marks.**

**Question B1**

- a) When  $x^3 - cx^2 - 4x + 6$  is divided by  $(x + 2)$  the remainder is 3 times larger than the remainder when  $2x^3 - x^2 + c^2x - 4$  is divided by  $(x - 1)$ .

Use the Remainder Theorem to find the values of  $c$ . **[ 4 ]**

- b) i. Write the expression  $x^2 - 16x + 57$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers. **[ 2 ]**

ii. **Use your result from part i** to solve the equation  $x^2 - 16x + 57 = 0$  giving your answers in exact surd form. **[ 2 ]**

iii. Sketch the graph of  $y = x^2 - 16x + 57$  (**this must not be done on graph paper**). Show clearly the coordinates where the curve crosses the  $x$  - and  $y$  - axes, and the coordinates of any stationary values. **[ 4 ]**

- c) A large retail outlet opens on 1 March and 6 people are taken on to work that day. On each subsequent day, 4 more people than the previous day are taken on i.e. on 1 March 6 are taken on; on 2 March 10 are taken on; on 3 March 14 are taken on; and so on.

i. On which day are 70 people taken on? **[ 2 ]**

ii. Find the total number of people who have been taken on by the end of March (31 days). **[ 3 ]**

- d) The sum of the first 12 terms of a geometric series with common ratio  $-2$  is  $-8190$ .

Find the first term. **[ 3 ]**

**Question B2**

a) Solve

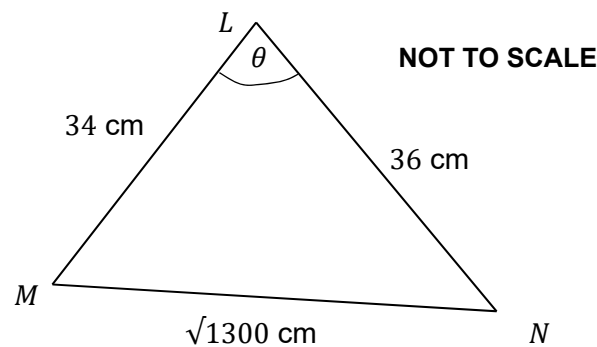
$$\frac{2m^3 \times 6m}{3m^7} = 2048 \quad [3]$$

b) Two variables,  $p$  and  $q$ , are connected by the formula

$$p = 24 e^{kq} - 36$$

where  $k$  is a constant.When  $q = 2$ ,  $p = 180$ .i. Show that  $k = \ln 3$ . Each stage of your working must be clearly shown. [3]ii. Find the value of  $p$  when  $q = 4$ . [2]iii. Find the value of  $\frac{dp}{dq}$  when  $q = 2.5$  [3]iv. Explain why  $p$  can never equal  $-40$ . [1]

c)

**Figure 1**Figure 1 shows the acute-angled triangle  $LMN$ . $LM = 34$  cm,  $LN = 36$  cm,  $MN = \sqrt{1300}$  cm and angle  $L = \theta^\circ$ .i. Write down an expression for  $\cos \theta$ . Give your answer in the form  $\frac{m}{n}$  where  $m$  and  $n$  are integers. [3]ii. Without finding the value of  $\theta$ , show that  $\sin \theta = \frac{15}{17}$ . All working must be shown. [2]**Part c) continues on the next page.**

iii. Find the area of triangle  $LMN$ . **[ 2 ]**

iv. Write down the fraction  $\frac{\sin N}{\sin M}$  in its simplest form. **[ 1 ]**

**Section B continues on the next page.**

**Question B3**

- a) i. Evaluate

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin x \, dx.$$

*All working must be shown. Just giving the answer, even the correct one, will score no marks if this working is not seen.* **[ 2 ]**

- ii. Explain what your answer means.
- [ 1 ]**

- b) i. A curve has equation
- $y = 2x^3 - 9x^2 - 24x + 4$
- .

Find the coordinates of the stationary values on the curve. **[ 5 ]**

- ii. Determine, for each stationary value, whether it is a maximum or a minimum.
- [ 4 ]**

- c)

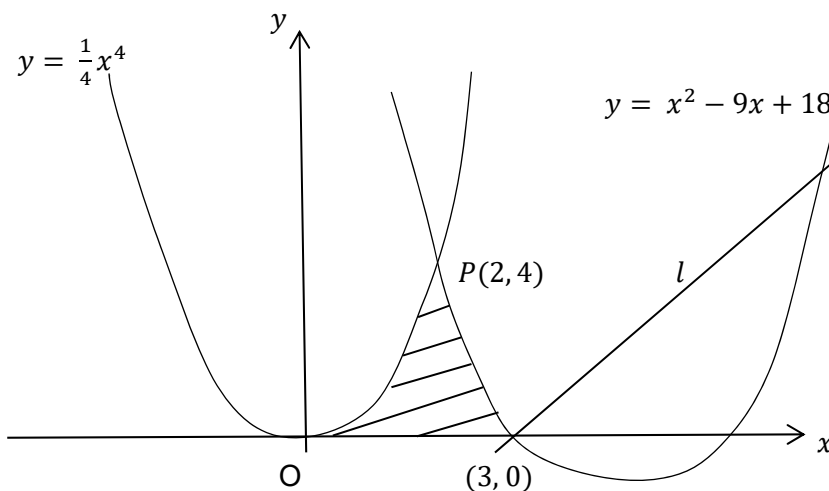
**Figure 2**

Figure 2 shows the curves  $y = \frac{1}{4}x^4$  and  $y = x^2 - 9x + 18$ . The curves intersect at point  $P(2, 4)$ . The line  $l$  is also shown which is the normal to the curve  $y = x^2 - 9x + 18$  at the point  $(3, 0)$ .

- i. Find the area, which is shaded on the diagram, that is bounded by both curves and the  $x$  - axis. **[ 5 ]**
- ii. Find the equation of line  $l$  **[ 3 ]**

**Question B4**

- a) The times that 80 customers spend in a department store are recorded and the results are shown in the table below.

<b>Time, <math>t</math>, in minutes</b>	<b>Frequency</b>
$0 < t \leq 10$	3
$10 < t \leq 20$	7
$20 < t \leq 30$	12
$30 < t \leq 40$	22
$40 < t \leq 50$	19
$50 < t \leq 60$	12
$60 < t \leq 70$	5

(You may wish to copy and extend this table to help you answer some of the questions below.)

- i. Estimate the mean. **[ 3 ]**
- ii. On graph paper, draw a cumulative frequency curve. **[ 4 ]**
- iii. Use your cumulative frequency curve to estimate the median and interquartile range. **[ 3 ]**
- iv. Does the distribution seem to be skewed? Give a reason. **[ 2 ]**
- b) In a group of 50 students, 20 study Biology, 29 study Chemistry and 13 study neither Biology nor Chemistry.
- i. Show this information on a Venn diagram. **[ 3 ]**
- ii. A student is selected at random. If  $B$  denotes the event 'the student studies Biology' and  $C$  denotes the event 'the student studies Chemistry', write down  $p(B')$ ,  $p(B \cap C')$  and  $p(B' \cup C')$ . **[ 3 ]**
- iii. Investigate whether events  $B$  and  $C$  are independent. **[ 2 ]**

**Section B continues on the next page.**



**Question B5**

- a) A student invested £2000. At the end of 6 years, the compound interest was £388. Find the rate of interest. [ 4 ]

- b) It is claimed that a certain chemical, when added to the soil, will increase the number of carrots grown. A gardener planted equal amounts of carrot seed in 5 plots and added different amounts of chemical to the soil in each plot. The numbers of carrots in each plot were then recorded and the results are shown in the table below.

Amount of chemical added in kg ( $x$ )	Number of carrots, in hundreds to the nearest hundred ( $y$ )
48	18
33	13
54	21
21	12
39	16

The data can be summarised as follows:

$$\sum x = 195 ; \quad \sum y = 80 ; \quad \sum x^2 = 8271 ; \quad \sum y^2 = 1334 ; \quad \sum xy = 3303.$$

- i. Find  $s_x$ ,  $s_y$  and  $s_{xy}$ . [ 3 ]
- ii. Hence find the product moment correlation coefficient. [ 1 ]
- iii. On these results, is there evidence that the claim can be supported? Give a reason. [ 2 ]
- c) In a town, 45% of the people are aged 50 or above. A random sample of 20 people is selected. Find the probability that
- i. 8 or less people are aged 50 or above. [ 1 ]
- ii. more than 10 people are aged 50 or above. [ 2 ]
- iii. exactly 11 people are aged **below** 50. [ 3 ]

**Part c) continues on the next page.**

- d) A discrete random variable,  $X$ , has probability distribution as given in the table below.

$x$	0	1	2	4	7
$p(X = x)$	0.15	0.1	0.25	0.3	0.2

You are given  $E(X) = 3.2$

- i. Find  $\text{Var}(X)$ . **[ 3 ]**

Another random variable,  $Y$ , is defined as  $Y = 3X - 1$ .

- ii. Write down  $E(Y)$  and  $\text{Var}(Y)$ . **[ 1 ]**

**Section B continues on the next page.**

**Question B6**

a) Find  $\frac{dy}{dx}$  in each of the following:

i.  $y = \cos^4 x$  **[ 2 ]**

ii.  $y = \tan x \cos^4 x$  **[ 2 ]**

iii.  $3x + 5x^2 + y^3 - 8y = 0$ , giving  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . **[ 3 ]**

b) i. Express  $\frac{x^2 - 3x - 2}{x^2(x - 2)}$  in the form  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 2}$  where  $A$ ,  $B$  and  $C$  are constants to be determined. **[ 4 ]**

ii. Hence evaluate

$$\int_3^4 \frac{x^2 - 3x - 2}{x^2(x - 2)} dx.$$

Give your answer in the form  $\ln\left(\frac{p}{q}\right) + \frac{1}{r}$  where  $p, q$  and  $r$  are integers.

*All working must be shown. An answer, even a correct one, will receive no marks if this working is not seen.* **[ 5 ]**

c) Use the substitution  $u = 3 + e^{2x}$  to find

$$\int e^{2x}(3 + e^{2x})^5 dx.$$

*All working must be shown.* **[ 4 ]**

**This is the end of the examination.**

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