NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination
2017-18

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Solve the simultaneous equations $8c + 3d = 7$

$$-4c + 4d = 13$$ **[4]**

Question A2

A box holds 3 red beads and 2 green beads. Two beads are taken from the box, one after the other and with no replacement.

Find the probability that both beads are of the same colour.   **[4]**

Question A3

When $3x^2 - 2x + kx - 11$ is divided by $(x - 4)$, the remainder is $3k$.

Use the Remainder Theorem to find the value of k.   *Show all working.*   **[3]**

Question A4

Expand $(2x - 3)^4$.   Give your answer in its simplest form and show all of your working.   **[4]**

Question A5

Solve the equation
$$3^{2x} - 10\,(3^x) = -9$$ **[4]**

Question A6

Solve the equation $5 \cos \theta = -4$ for $0 \leq \theta \leq \pi$.

Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures.   **[3]**

Question A7

Find

$$\int \frac{3x^3 - 2}{x} \, dx.$$

[3]

Question A8

The temperatures (in °C) are recorded on 8 days. The readings are:

$$6, \quad 0, \quad 2, \quad -1, \quad 3, \quad 8, \quad 4, \quad 2.$$

Find the mean, median and range.

[3]

Question A9

The numbers of cars sold at a garage were recorded over a three week period.

The results are shown in the table below.

Week	Number of cars sold	3-point moving average
1	$p + 5$	
2	$4p - 1$	$p^2 + 2$
3	$3p + 5$	

Find the value of p.

[3]

Question A10

A discrete random variable, X, has probability distribution as given in the table below.

x	-1	0	1	2	3
$p(X = x)$	0.1	0.15	0.2	0.25	0.3

Find the mean and the standard deviation of X.

[5]

Question A11

Find the coordinates of the points where the curve

$$y = \frac{x^2 + 3}{x + 1}$$

has stationary values. **[5]**

Question A12

Use the substitution $u = x^3 + 1$ to evaluate

$$\int_0^2 x^2 \sqrt{x^3 + 1} \, dx.$$

All working must be shown. An answer, even a correct one, will receive no marks if this working is not seen. **[4]**

Section B begins on the following page.

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a) A line passes through point $A(-1,\ 24)$ and point $B(8,\ 12)$.

Find the equation of the line which is perpendicular to AB and passes through point B. **[3]**

b) i. Express $x^2 + 8x + 11$ in the form $(x + a)^2 + b$. **[2]**

ii. Sketch the curve $y = x^2 + 8x + 11$.

Show clearly the coordinates of the stationary value and where the curve crosses the $y-$ axis. You do <u>not</u> need to show where the curve crosses the $x-$ axis. **[3]**

c) A geometric series is defined as $8 + 12 + 18 + \cdots$

i. Find the 7th term. **[2]**

ii. Find how many terms are needed for the sum of the series to go above one million. **[4]**

iii. Explain why there is no sum to infinity for this series. **[1]**

d) The function $f(x)$ is defined as $f(x) = 2x^3 - 5x^2 - 54x - 63$.

i. Divide $f(x)$ by $(x - 7)$. **[3]**

ii. Hence factorise $f(x)$ completely. **[1]**

iii. Solve $f(x) = 0$. **[1]**

Question B2

a) A farmer plants a field of wheat. The wheat is tested each day for moisture content. The first test shows a moisture content of 30%.

It is believed that the moisture content, C, and the number of days after the first test, t, are connected by the formula

$$C = Ae^{-0.03t}$$

where A is a constant.

i. Show that $A = 30$. [1]

ii. Find the moisture content 4 days after the first test. [2]

The wheat is ready for harvesting when the moisture content is 15%.

iii. After how many days from the first test can harvesting begin? [3]

b) Solve the equation

$$2\log_2(x + 6) - \log_2(x^2 + 5x - 6) = 3 \qquad (x > 1)$$

Each stage of your working must be clearly shown. [5]

c) You are given $\sin\theta = \dfrac{12}{37}$. <u>Without</u> working out the value of θ, find $\cos\theta$

giving your answer in the form $\dfrac{m}{n}$ where m and n are integers. [2]

All working must be shown.

Part d) is on the next page.

Question B2 – (continued)

d)
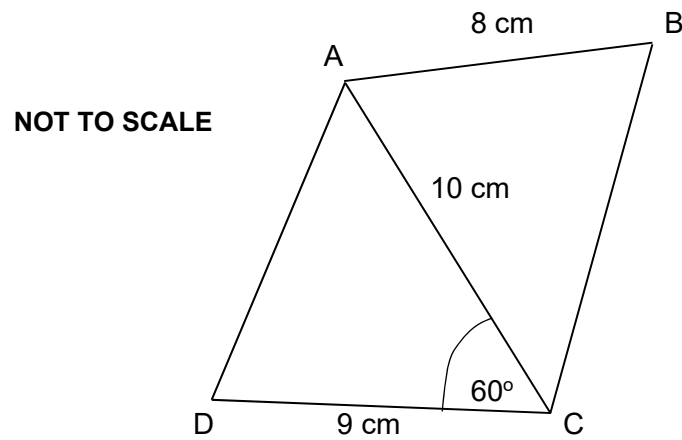

Figure 1

Figure 1 shows the quadrilateral ABCD which is made up of two acute-angled triangles ABC and ACD. AB = 8 cm, AC = 10 cm, DC = 9 cm and angle ACD = 60°.

i.   Find AD.   **[3]**

ii.   Find angle ADC.   **[2]**

iii. The area of triangle ACD is the same as the area of triangle ABC.
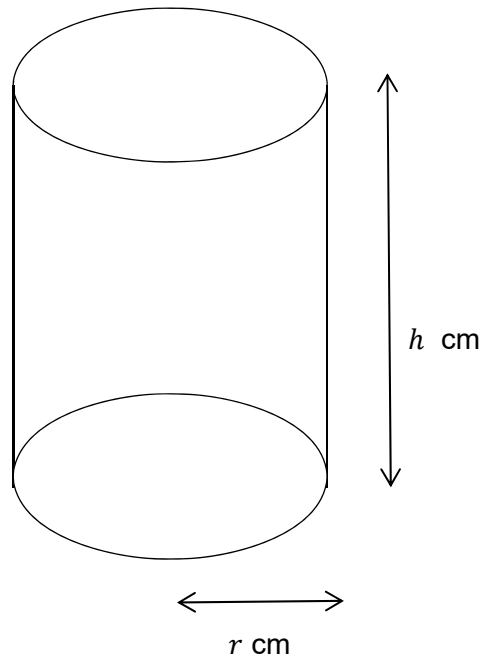
      Find angle BAC.   **[2]**

Question B3

a)

Figure 2

Figure 2 shows a **solid** cylinder of radius r cm and height h cm.

The total surface area is 294π cm².

i.   Find h in terms of r.   **[2]**

ii. Show that the volume of the cylinder, V, is given by

$$V = 147\pi r - \pi r^3 \, .$$ **[3]**

iii. Use $\frac{dV}{dr}$ to find the value of r which gives the maximum volume.   **[4]**

iv. Confirm that your value of r gives a maximum.   **[3]**

Part b) is on the next page.

Question B3 – (continued)

b)

$$y = 3 + 2x - x^2$$
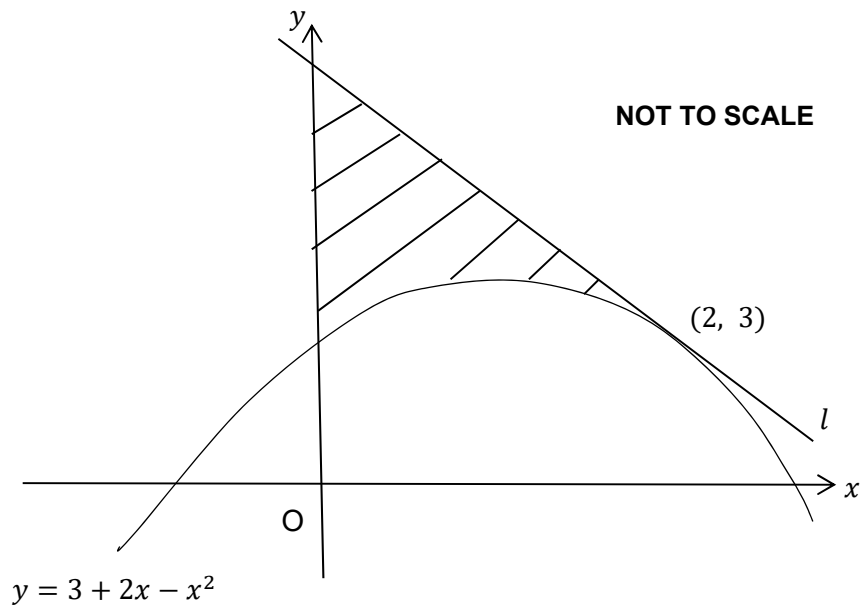
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(2, 3)

l

Figure 3

Figure 3 shows the curve $y = 3 + 2x - x^2$ and line l which is a tangent to the curve at the point $(2, 3)$.

i. Find the equation of line l. Give your answer in the form $y = mx + c$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = 3 + 2x - x^2$, line l and the $y -$ axis. **[5]**

All working must be shown.

Question B4

a) The table below shows the numbers of a certain type of mobile telephone sold in a shop over a 28 day period during the summer of 2017.

Number of telephones sold	Frequency
0 – 4	2
5 – 9	6
10 – 14	8
15 – 19	7
20 - 24	5

(You may wish to copy and extend this table to help you answer some of the questions below.)

i.   Estimate the mean and standard deviation.   **[6]**

ii. Explain why your answers to part i are only estimates.   **[1]**

iii. In which interval does the upper quartile lie?   **[1]**

iv. Can your results be used to predict the sales during the same 28 day period in the summer of 2018? Give a reason.   **[2]**

b) Two events, A and B, are such that $p(A) = 0.3$, $p(A \cup B) = 0.72$ and $p(A \cap B) = 0.18$

i.   Show that $p(B) = 0.6$ *All working must be shown*.   **[2]**

ii. Draw a Venn diagram to show these probabilities.   **[3]**

iii. Hence work out $p(A' \cup B)$, $p(A \cap B')$ and $p(A|B)$.   **[3]**

iv. Investigate whether events A and B are independent.   **[2]**

Question B5

a) A student invested £P on 1 January 2016. This earned compound interest at a rate of 2.5%.

   On 1 January 2017, the amount was £1271.

       i.   Find the value of P.   **[2]**

       ii.   If the rate stays the same, find the expected amount on 1 January 2021.   **[2]**

       iii.   Work out the expected **total** interest earned over the five years from 1 January 2016 until 1 January 2021.   **[1]**

b) A department store sells hats and coats. A record is kept of the numbers of sales of each over a six month period. The results are shown in the table below.

Number of hats sold (x)	Number of coats sold (y)
48	57
36	40
76	64
62	53
25	74
17	42

   The data can be summarised as follows:

   $\sum x = 264;$ $\sum y = 330;$ $\sum x^2 = 14134;$ $\sum xy = 14890.$

       i.   Find s_x^2 and s_{xy}. Hence find the equation of the regression line of y on x. Give your answer in the form $y = mx + c$. *Show all working.*   **[4]**

       ii.   Use your equation to predict the value of y when $x = 60$.   **[2]**

       iii.   Is your prediction in part ii reliable? Give a reason.   **[1]**

c) The masses of apples are assumed to follow a Normal distribution with mean 110 grams and standard deviation 10 grams.

       i.   10% of apples are below x grams. Find the value of x.   **[3]**

       Part c) continues on the next page.

ii.   If an apple has mass 117 grams or more, it is classified as 'large'.

Find the probability that an apple, selected at random, is 'large'.   **[3]**

iii.   The apples are packed in boxes of 40. A box is selected at random.

Find the probability that the box contains exactly 7 'large' apples.   **[2]**

Question B6

a)   A curve has equation $4x - x^2y + y^3 = 1$.

i.   Find $\frac{dy}{dx}$ in terms of x and y.   *All working must be shown.*   **[4]**

ii.   When there is a stationary point on the curve, y and x are connected by the equation $y = \frac{c}{x}$.   State the value of c.   **[1]**

b)   Differentiate $y = (4x^2 - 3x + 1)^6$.   **[2]**

c)   i.   Express $\frac{4x + 5}{(x - 1)(x + 2)}$ in the form $\frac{A}{x - 1} + \frac{B}{x + 2}$ where A and B are

constants to be determined.   **[3]**

ii.   Hence evaluate

$$\int_{2}^{3} \frac{4x + 5}{(x - 1)(x + 2)} \ dx.$$

Give your answer in the form $\ln k$ where k is an integer.

All working must be shown. An answer, even a correct one, will receive no marks if this working is not seen.   **[5]**

d)   Use integration by parts to find

$$\int 9x^2 \, e^{-3x} \ dx.$$ **[5]**

All working must be shown.

This is the end of the examination.