NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination
2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Gradient = $-\frac{1}{3}$ **or** writes equation as $x + 3y + k = 0$ and substitutes in $x = 3;\ y = 7$ **[M1]**

Correct form of equation $[\, y - 7 = \text{their gradient}(x - 3)]$ **or** finds a value for k **[M1]**

$x + 3y - 24 = 0$ (must be in this form) **[A1]**

Question A2

$\frac{5}{11}$, $\frac{4}{10}$ or $\frac{3}{9}$ seen **[M1]**

Multiplies their probabilities **[M1]**

$= \frac{2}{33}$ or anything rounding to 0.061 **[A1]**

Question A3

Substitutes $x = 2$ into expression [The Remainder Theorem must be used.] **[M1*]**

= 0 **[A1]**

It is a factor. **[B1]**

Question A4

$a \times (\frac{9}{10})^3 = a - 271$ **[M1]**

Rearranges and finds a value for a **[M1]**

$a = 1000$ **[A1]**

$S_\infty = \frac{\text{their } a}{1 - \frac{9}{10}}$ **[M1]**

$= 10000$ (Allow follow through) **[A1]**

Question A5

$\log_{10}25 + \log_{10}8 - \log_{10}2$ Uses the log power law at least once **[M1*]**

Uses the log addition or subtraction law **[M1*]**

$= \log_{10}100$ or $2\log_{10}10$ (Either version must be seen) **[A1]**

$= 2$ **[A1]**

Question A6

Uses the cosine formula $(16^2 = 14^2 + 15^2 - 2 \times 14 \times 15 \times \cos A)$ $[\cos A = \frac{165}{420}]$ **[M1]**

Calculates correctly and in the right order. **[M1]**

Angle A $= 66.8676\ldots$ (can be implied) **[A1]**

$= 66.9$ (degrees) to 3 significant figures (Allow follow through) **[A1ft]**

Special case: If 1.17 (radians) is seen, award 3 marks out of 4

Question A7

Integrates and sets equal to 35 (Integration must be used) **[M1*]**

Forms a quadratic equation $(2a^2 + 3a - 35 = 0)$ **[M1]**

Factorises or uses formula $[(2a-7)(a+5) = 0$ or $a = \frac{-3 \pm\sqrt{[3^2 - 4 \times 2 \times -35]}}{2 \times 2}]$ **[M1]**

$a = -5, \ \frac{7}{2}.$ **[A1]**

Question A8

Uses $p(A|B) = \frac{p(A \cap B)}{p(B)}$ to find $p(A \cap B)$ $(= \frac{1}{2})$ and then uses $p(B|A) = \frac{p(A \cap B)}{p(A)}$ to find a value for $p(A)$ **[M1]**

$p(A) = \frac{3}{5}$ **[A1]**

Uses $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ to find $p(A \cup B)$ **[M1]**

$p(A \cup B) = \frac{9}{10}$ (allow follow through, provided the answer is between 0 and 1) **[A1ft]**

Accept equivalent fractions, decimals and percentages.

Question A9

For X: weak negative correlation **[B1]**

For Y: strong positive correlation **[B1]**

For Z: virtually no correlation or very weak negative correlation **[B1]**

Allow equivalent descriptions before the positive or negative and be on the generous side. If 'very weak' is used for X, this is all right but it cannot be used again for Z, and vice versa.

Question A10

$200 \pm \frac{6 \times 1.96}{\sqrt{9}}$ **[M1]**

Anything rounding to 196 **(A1)** and 204 **(A1)** (cm) **[A2]**

Question A11

Uses the Quotient Rule or the Product Rule [($\frac{dy}{dx}$ =) $\frac{4(x+3)-(4x+2)}{(x+3)^2}$ or $-(4x+2)(x+3)^{-2}+4(x+3)^{-1}$] **[M1*]**

Sets equal to $\frac{2}{5}$, and solves to find at least one value of x $(-8 \text{ and } 2)$ **[M1]**

Substitutes their $x-$ values (there must now be two of them) into the original expression to find the $y-$ values. **[M1]**

$(2,\ 2)$; $(-8,\ 6)$ **[A1]**

Question A12

Uses integration by parts in the right direction **[M1*]**

$= 16x^2 \times \frac{1}{2}e^{2x}$ **(A1)** $- \int 32x \times \frac{1}{2}e^{2x}\ dx$ (A1) for first part **[A1]**

Applies integration by parts again **[M1*]**

$32x \times \frac{1}{4}e^{2x}$ **(A1ft)** $- \int 32 \times \frac{1}{4}e^{2x}\ dx$ Do not worry about signs here. Allow follow through from the integration in the second line **[A1ft]**

$8x^2e^{2x} - 8xe^{2x} + 4e^{2x} + c$ (No follow through) **[A1]**

Section B

Question B1

a) Point X lies at (-6, 0) and point Y lies at (0, -8) **[M1]**

Uses Pythagoras **[M1]**

= 10 (units) **[M1]**

b) $y = 2x - 7$ or $x = \frac{y+7}{2}$ **[M1*]**

Substitutes into second equation **[M1*]**

Reaches a quadratic equation $[2x^2 - 7x + 3 = 0 \text{ or } y^2 + 7y + 6 = 0]$ **[M1*]**

Factorises or uses formula $[(2x-1)(x-3) = 0 \text{ or } x = \frac{7 \pm \sqrt{[(-7)^2 - 4 \times 2 \times 3]}}{2 \times 2}$ **[M1]**
$(y+1)(y+6) = 0 \text{ or } y = \frac{-7 \pm \sqrt{[7^2 - 4 \times 1 \times 6]}}{2 \times 1}]$

Substitutes their x or y values into the original equation **[M1]**

$\left(\frac{1}{2}, -6\right)$ and $(3, -1)$ Answers do not have to be written as coordinates but it must be clear that they have been matched up correctly. **[A1]**

c) i. $x > 4$ **[B1]**

ii. $x \geq -6$ **(B1)*** $x \leq 6$ **(B1)*** or $-6 \leq x \leq 6$ **(B1)** for each end **[B2]**
*Please note: if this version of the answers is quoted, the two ranges can be separated by a space, a comma or the word 'and'. The final mark is lost if the word 'or' is seen.

iii. 5, 6 **[B1]**

d) i. $a + (12 - 1) \times 6 = 16$ **[M1]**

$a = -50$ **[A1]**

ii. $S_{35} = \frac{35}{2}\,[\,2 \times \text{their } a + (35 - 1) \times 6]$ **[M1]**

Calculates correctly in the right order **[M1]**

= 1820 **[A1]**

e) ${}^{10}C_4 \times 2^6 \times (\frac{1}{2})^4$ Allow ${}^{4}C_{10}$ for ${}^{10}C_4$ and the presence of x **[M1]**

= 840 **[A1]**

Question B2

a) i. $20 = 320\left(2^{2k}\right) + 15$ and rearranges to reach $2^{2k} = \cdots$ $\left(\frac{5}{320}\right)$ **[M1]**

Takes logs correctly and reaches $2k = \cdots$ $\left[\log\left(\frac{5}{320}\right)\right]$ **[M1]**

$k = -3$ **[A1]**

ii. Substitutes $x = \frac{2}{3}$ into the formula with their k **[M1]**

95 **[A1ft]**

iii. 15 **[B1]**

b) i. $x = 8$ **[B1]**

ii. $\log_4\left[\frac{3x}{x+4}\right] = \log_4 2$

Correctly combines the logs **[M1*]**

Adapts RHS and removes logs at the right time **[M1*]**

Solves **[M1]**

$x = 8$ **[A1]**

c) $\frac{1}{2} \times (a+4)(2a+1) \times \sin 60 = 15\sqrt{3}$ **[M1]**

Forms a quadratic equation ($2a^2 + 9a - 56 = 0$) **[M1]**

Factorises or uses formula **[M1]**
$\left[\ (2a-7)(a+8) = 0 \text{ or } a = \frac{-9 \pm \sqrt{[9^2 - 4 \times 2 \times -56]}}{2 \times 2}\right]$

$a = \frac{7}{2}$ or equivalent (If the - 8 is also quoted, this mark is lost. Placing it in brackets is good enough to indicate non-inclusion.) **[A1]**

d) $\sin(\theta + 50) = \pm\frac{1}{2}$ **[M1]**

$\theta + 50 = 30,\ 150,\ 210,\ 330$ At least one correct **[M1]**

Subtracts 50 at the right time **[M1]**

$\theta = 100,\ 160,\ 280,\ 340.$ **(A1)** for any two correct; **(A2)** for all correct **[A2]**
Ignore extra solutions outside the range but take off one mark for any extra solutions in the range. [If the negative sign is missed in the first line, it is possible to score M0 M1 M1 A1 A0]

Question B3

a) i. $1536 = 2x \times 4x + 2 \times 4x \times h + 2 \times 2x \times h$ **[M1]**

$h = \frac{1536 - 8x^2}{12x}$ or $\frac{128}{x} - \frac{2x}{3}$ **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

$V = 2x \times 4x \times h$ **[M1*]**

Substitutes in their h **[M1*]**

$V = 1024x - \frac{16x^3}{3}$ Result reached with no errors seen. **[A1]**

iii. Attempts to differentiate (sight of the constant or x^2 term is sufficient) $[\frac{dV}{dx} = 1024 - \frac{48x^2}{3}]$ **[M1*]**

Sets equal to 0 (this can be implied) **[M1]**

Solves to reach $x^2 = \cdots$ (64) **[M1]**

$x = 8$ (cm) [This mark is lost if -8 is also included. Placing it in brackets is good enough to indicate non-inclusion.] **[A1]**

iv. Finds second derivative (Presence of x term is sufficient for this mark) **[M1*]**

$\frac{d^2V}{dx^2} = -\frac{96x}{3}$ Correct answer **[A1]**

This is negative (when $x = 8$) so there is a maximum (reason and conclusion). Allow follow through for their $\frac{d^2V}{dx^2}$ provided it is negative. **[A1ft]**

or Takes a numerical value between 0 and 8 and shows $\frac{dV}{dx} > 0$ **(M1*)**

Takes a numerical value above 8 and shows $\frac{dV}{dx} < 0$ **(M1*)**

Thus there is a maximum (at $x = 8$) **(A1ft)**
Allow follow through on their $\frac{dV}{dx}$ provided it gives a maximum.

Part b) is on the next page.

Question B3 - (continued)

b) i. Subtracts equations before integrating or subtracts their areas having integrated separately. **[M1]**

Attempts to Integrate $\int_0^{\ln 2}(e^x - e^{-x} + 2)\ dx$ or their two separate integrals. Sight of the x attached to the constant term is sufficient. **[M1*]**

Substitutes limits into their integrated expression(s) and subtracts the right way round **[M1]**

$\frac{1}{2} + \ln 4$ or equivalent, or anything rounding to 1.89 **[A1]**

ii. $\frac{dy}{dx} = e^x$ **[M1*]**

Substitutes in $x = \ln 4$, inverts and changes sign $(-\frac{1}{4})$ **[M1]**

$y - 7 = -\frac{1}{4}(x - \ln 4)$ or equivalent **[A1]**

iii. $\frac{dy}{dx}\ (e^x) \neq 0$ so there are no stationary values (or similar explanation). **[B1]**

Question B4

a) i. Readings are discrete **(B1)** with reason **(B1)**. Examples could be 'readings take distinct values only'; 'readings are confined to whole numbers'; 'data can be counted as opposed to measured'; etc. **[B2]**

ii.

Mid-value (x)	**Frequency** (f)	$\boldsymbol{x \times f}$	$\boldsymbol{x^2 \times f}$
2	19	38	76
7	14	98	686
13	9	117	1521
20	5	100	2000
28	3	84	2352

$\sum(x \times f) = 437 \quad \sum(x^2 \times f) = 6635$

Completes $x \times f$ column **[M1]**

Divides their $(x \times f)$ by 50 **[M1]**

$= 8.74$ **[A1]**

Completes $x^2 \times f$ column **[M1]**

Divides their $\sum(x^2 \times f)$ by 50, subtracts square of their mean and takes square root **[M1]**

= anything rounding to 7.5 **[A1]**

iii. $5 - 9$ **[B1]**

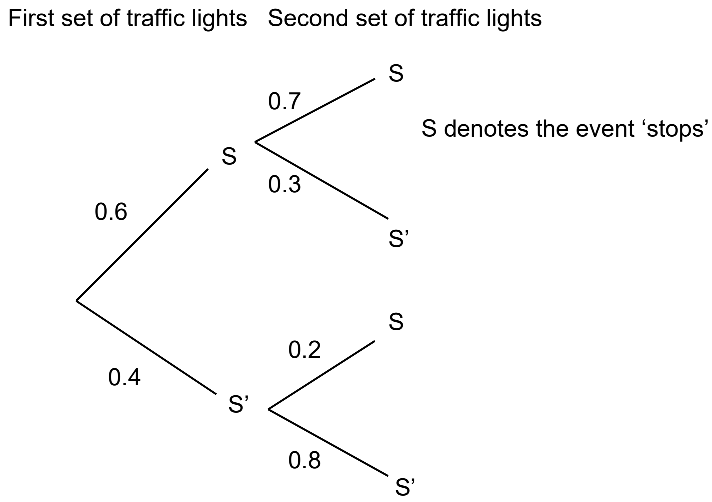
iv. $\frac{8}{50} \times \frac{7}{49}$ **[M1]**

$= \frac{4}{175}$ or anything rounding to 0.023 **[A1]**

Part b) is on the next page.

Question B4 - (continued)

b) i.



(B1) for correct first set of branches; **(B1)** for correct second set **[B2]**

ii. $1 - p$(not stopping at both) $[1 - \text{their}(0.4 \times 0.8)]$ **[M1]**
(or any other valid method)

$= 0.68$ **[A1]**

iii. Multiplies along correct branches ($0.6 \times 0.7;\quad 0.4 \times 0.2$) **[M1]**

Adds **[M1]**

$= 0.5$ **[A1]**

iv. $\frac{0.6 \times 0.7}{\text{their part iii}}$ **[M1]**

$= 0.84$ Allow follow through on their part iii **[A1ft]**

Question B5

a) i.

$s_x = \sqrt{30} \approx 5.48$ **[B1]**

$s_y = \sqrt{247} \approx 15.7$ **[B1]**

$s_{xy} = 58$ **[B1]**

ii. PMCC = anything rounding to 0.67 **[B1ft]**

b) i. $a = 148$ **[B1]**

$b = 168$ **[B1]**

ii. No **(B1*)** plus reason **(B1)** such as 'readings taken over too short a time interval'; 'in 6 months' time it will be in another season and sales may be affected'; etc.
*This mark is only given if a reason, even a wrong one, follows. **[B2]**

c) i. $5100 \div \frac{85}{100}$ or similar working **[M1]**

6000 (pounds) **[A1]**

ii. $5100 \times (\frac{9}{10})^3$ **[M1]**

$= £3717.90$ (Allow anything rounding to $£3720$) **[A1]**

iii. $\frac{\text{their i} - \text{their ii}}{\text{their i}} \times 100$ **[M1]**

= anything rounding to 38% **[A1]**

d) i. Anything rounding to 0.217 **[B1]**

ii. $1 - p($ 7 or less goals$)$ $(1 - 0.7869)$ **[M1]**

= anything rounding to 0.213 **[A1]**

iii. $^{15}C_6 \times 0.4^6 \times 0.6^9$ or p(6 goals or less) - p(5 goals or less) **[M1]**

= anything rounding to 0.21 **[A1]**

iv. Independence assumed. **[B1]**

Question B6

a) i. $12 + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 6x = 0$ **[M1*]**

Attempt at implicit differentiation (sight of $y\frac{dy}{dx}$ or $5\frac{dy}{dx}$ is sufficient)

Gathers $\frac{dy}{dx}$ terms on to one side and factorises (this mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{6x - 12}{2y - 5}$ **[A1]**

ii. $(x =)\ 2$ **[B1]**

iii. Substitutes their x value into original expression and forms a quadratic equation $(y^2 - 5y - 6 = 0)$ **[M1]**

Factorises or uses formula **[M1]**

$[(y-6)(y+1) = 0$ or $y = \frac{5 \pm \sqrt{[(-5)^2 - 4 \times 1 \times -6]}}{2 \times 1}$

Coordinates are $(2,\ 6);\ \ (2, -1)$ **[A1]**

b) i. $4 + \frac{2x+5}{x^2 + 5x + 6}$ **[B1]**

ii. $2x + 5 = B(x+3) + C(x+2)$ **[M1]**

$B = 1;\quad C = 1$ **(A1)** for each **[A2]**

iii. Uses previous result $[\ \int_0^3 \left(4 + \frac{1}{x+2} + \frac{1}{x+3}\right) dx\ ]$ **[M1*]**

Attempts to integrate (sight of a log term or x attached to the constant term is sufficient for this mark $[4x + \ln(x+2) + \ln(x+3)]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Combines the logs using the addition law at any stage **[M1]**

$= 12 + \ln 5$ **[A1]**

[If any of the constants A, B or C is wrong, it is still possible to obtain 4 marks in this part.]

Part c) is on the next sheet.

Question B6 - (continued)

c) $du = \frac{1}{\cos^2 x}\, dx$ or equivalent **[M1*]**

Writes integral in terms of u ($\int_0^{\sqrt{3}} u^5 \; du$) [The limits do not need to have been changed for this mark] and attempts to integrate. **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. If the original limits are in place, the expression must be changed back into terms in x. **[M1]**

$= \frac{9}{2}$ or equivalent **[A1]**

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