NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business Examination 2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A5. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Mid-point is at $(2, -4)$ **[M1]**

Finds gradient of AB $(-\frac{1}{4})$ **[M1]**

Inverts and changes sign (4) **[M1]**

$y + 4 = 4(x - 2)$ or equivalent **[A1]**

Question A2

$\frac{4}{10}$ or equivalent seen **[B1]**

$\frac{3}{12}$ or equivalent seen **[B1]**

Multiplies **[M1]**

$\frac{1}{10}$ or equivalent **[A1]**

Question A3

$$
\begin{array}{rl}
 & 2x^2 + 5x - 3 \\
x - 2 \;\big| & 2x^3 + x^2 - 13x + 6 \\
 & \underline{2x^3 - 4x^2} \\
 & \quad 5x^2 - 13x \\
 & \quad \underline{5x^2 - 10x} \\
 & \qquad -3x + 6 \\
 & \qquad \underline{-3x + 6} \\
 & \qquad \quad \ldots\ldots\ldots
\end{array}
$$

Correct first division **[M1]**

Any correct subsequent division **[M1]**

Correct quotient **[A1]**

Question A4

a) $15a = a + (8-1)d$ **[M1*]**

At least one correct line of intermediate working to reach $d = 2a$ **[A1]**

b) Uses a correct version of the sum of n terms formula in terms of a or d **[M1]**

$[1600 = \frac{20}{2}[d + 19d]$ or $\frac{20}{2}(2a + 19(2a)]$

Calculates correctly and in the right order **[M1]**

$a = 4,\ d = 8$ **[A1]**

Question A5

Uses logs correctly $(x \log 6 = \log 1850)$ **[M1]**

$x = 4.1986\ldots$ (can be implied) **[A1]**

$= 4.20$ to three significant figures. Allow follow through for their x **[A1ft]**

Question A6

$\frac{1}{2} \times 7.2 \times 6.8 \times \sin 54\ (= 19.8)$ **[M1]**

$\frac{1}{2} \times 8.6 \times 7.9 \times \sin 62\ (= 30)$ **[M1]**

Adds **[M1]**

= anything rounding to 49.8 (cm^2) **[A1]**

Question A7

$-\frac{3}{x^2} - \sin x + 3e^{-3x}$ Accept equivalent expressions. **(B1)** for each **[B3]**

Question A8

Attempts to integrate (sight of $\ln x$ or an $x -$ term is sufficient for this mark) **[M1*]**
$(\ln x - 2x)$

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

$= \ln 2 - 6$ (Must be in this form) **[A1]**

Question A9

$p(A \cup B) = \frac{1}{4} + \frac{3}{5} - \frac{1}{8}$ **[M1]**

$= \frac{29}{40}$ or equivalent **[A1]**

$p(B|A) = \frac{1}{8} \div \frac{1}{4}$ **[M1]**

$= \frac{1}{2}$ or equivalent **[A1]**

Question A10

$2000(1 + \frac{5}{100})^3 \ (= 2315.25)$ **[M1]**

Their $2315.25(1 + \frac{4}{100})^2 \ (= 2504.17 \ldots)$ **[M1]**

Subtracts 2000 from their final amount **[M1]**

Anything rounding to £504 **[A1]**

Question A11

$^{20}C_{13} \times 0.7^{13} \times 0.3^7$ **(M1)** $^{20}C_{14} \times 0.7^{14} \times 0.3^6$ **(M1)** **[M2]**

Adds **[M1]**

Anything rounding to 0.356 **[A1]**

or calculates 7 or less yellow and 5 or less yellow, and uses tables

$p(7 \text{ or less}) \ [0.7723]$, $p(5 \text{ or less}) \ [0.4164]$ **(M1) + (M1)**

Subtracts **(M1)**

Anything rounding to 0.356 **(A1)**

Question A12

$du = 4x^3 \, dx$ or equivalent **[M1*]**

Writes integral in terms of u $[\frac{1}{4} \times \frac{1}{u^3}]$ and attempts to integrate **[M1*]**
(The limits do not have to be changed)

Substitutes limits into their integrated expression and subtracts right way round. **[M1]**
If limits have not been changed, expression must be put back into terms in x.

$\frac{3}{32}$ or equivalent but must be in this form. **[A1]**

Section B

Question B1

a) i. Solves the inequality **[M1]**

$x < 5$ **[A1]**

ii. Factorises or uses formula **[M1]**

$[(x+7)(x-2) = 0 \text{ or } x = \frac{-5 \pm \sqrt{[5^2 - 4 \times 1 \times -14]}}{2 \times 1}]$

Finds two critical values $(-7,\ 2)$ **[M1]**

$x \leq -7$ **(A1)** $x \geq 2$ **(A1)** **[A2]**

Please note: the two ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen.

iii. 2, 3, 4 **[B1]**

b) *Please note: the Remainder Theorem must be used*.

Sets $x = -k$ and substitutes into expression **[M1*]**

Sets equal to $-3k$ and forms a quadratic equation **[M1*]**

Factorises or uses formula $[(k-5)(k+1) = 0 \text{ or } k = \frac{4 \pm \sqrt{[(-4)^2 - 4 \times 1 -5]}}{2 \times 1}]$ **[M1]**

$k = -1,\ 5.$ **[A1]**

c) ${}^7C_3 \times p^4 \times (-2)^3$, ${}^7C_4 \times p^3 \times (-2)^4$ M1 for either. Allow ${}^xC_y$ for ${}^yC_x$ and the presence of x **[M1]**

Sets expressions equal to each other and reaches a value for p. There must now be no x present. **[M1]**

$p = -2$ [Ignore any reference to $p = 0$] **[A1]**

d) ii. $ar^6 \div ar = 364\frac{1}{2} \div 1536$ (either way round) **[M1]**

Reaches r^5 or $r^{-5} = \cdots$ $(\frac{243}{1024}$ or $\frac{1024}{243})$ **[M1]**

$r = \frac{3}{4}$ **[A1]**

Substitutes their r into either expression **[M1]**

$a = 2048$ **[A1]**

iii. $S_\infty = 8192$ (this must be seen), so series will not reach 8200. **[A1ft]**

Question B2

a) i. 100 **[B1]**

ii. Substitutes $t = 6$ into formula **[M1]**

Anything rounding to 274 **[A1]**

iii. Rearranges correctly and reaches $e^{0.2t} = \cdots$ $(\frac{167}{75})$ **[M1]**

Takes logs and reaches $0.2t = \cdots$ $[\ln\left(\frac{167}{75}\right)]$ **[M1]**

$t =$ anything rounding to 4 (years) **[A1]**

iv. Attempts to differentiate (sight of $ke^{0.2t}$ is sufficient but $kte^{0.2t}$ is M0) **[M1]**

Substitutes $t = 10$ into their $\frac{dN}{dt}$ **[M1]**

Anything rounding to 111 **[A1]**

v. 'The number of birds increases at a rate of 111 per year after 10 years' or similar words. 10 years should be mentioned and an indication of 'rate' should be present: 'birds per year' is good enough. Allow follow through on their answer to part iv. **[B1ft]**

b) Uses sine formula to find PR $(\frac{\text{PR}}{\sin 69} = \frac{15}{\sin 51})$ **[M1]**

Finds a value for PR (= 18) **[M1]**

Uses the sine or cosine formula to find QR $(\frac{\text{QR}}{\sin 60} = \frac{15}{\sin 51}$ or $\frac{\text{their PR}}{\sin 69})$ **[M1]**

(or $\text{QR}^2 = 15^2 + \text{their PR}^2 - 2 \times 15 \times \text{their PR} \times \cos 60$)

Finds a value for QR (= 16.7) **[M1]**

Perimeter = anything rounding to 49.7 (cm) **[A1]**
[The marks can be given for any other valid solution]

c) $2\theta =$ any one of answers rounding to 112, 248, 472, 608 **[M1]**

Realises the range for 2θ extends from 0 to 720 degrees **[M1]**

Divides by 2 at the right time **[M1]**

$\theta =$ anything rounding to 56, 124, 236, 304 **(A1)** any 2 correct **(A2)** all correct. One mark lost for extra solutions in range: ignore solutions outside range. **[A2]**

Question B3

a) i. $V = x(90 - 2x)(120 - x)$ **[M1*]**

At least one intermediate line of working

Reaches $V = 2x^3 - 330x^2 + 10800x$ with no errors seen **[A1]**

ii. Attempts to differentiate (sight of x^2, x or 10800 is sufficient) **[M1*]**
$[\frac{dV}{dx} = 6x^2 - 660x + 10800\ ]$

Sets equal to 0 (can be implied) and forms a quadratic equation **[M1]**
$[\ 6(x^2 - 110x + 1800) = 0]$
Factorises or uses formula
$[6(x - 20)(x - 90) = 0 \text{ or } x = \frac{110 \pm\sqrt{[(-110)^2 - 4 \times 1 \times 1800]}}{2 \times 1}\ ]$ **[M1]**

$x = 20$ (ignore the 90 if it is also quoted) **[A1]**

iii. Attempts to differentiate again (sight of $x-$ term or the constant is sufficient) **[M1*]**
$\frac{d^2V}{dx^2} = 12x - 660$ Correct answer **[A1]**

This is negative (when $x = 20$) so there is a maximum

[If the 90 has not been discarded earlier, then a comment that this is positive when $x = 90$ and is thus a minimum must also be seen, along with the inclusion of $x = 20$ in the previous sentence] (Allow follow through on their $\frac{d^2V}{dx^2}$ provided it is negative). **[A1ft]**

or Takes a numerical value of x between 0 and 20 and shows $\frac{dV}{dx} > 0$ **(M1*)**
Takes a numerical value of x greater than 20 and shows $\frac{dV}{dx} < 0$ **(M1*)**

Thus there is a maximum when $x = 20$ (Allow follow through on their x and their $\frac{dV}{dx}$ provided it gives a maximum) **(A1ft)**

[If the 90 has not been discarded earlier, then the second M1 can only be given if there is also an inclusion of taking a numerical value greater than 90 and showing that $\frac{dV}{dx}$ is again positive, so there is a minimum at $x = 90$.]

Special case: if in part iii the candidate works with $x^2 - 110x + 1800$, full marks can still be achieved.

iv. 100000 (cm^3) (Allow follow through on their x) **[A1ft]**

Part b) is on the next page.

Question B3 - (continued)

b) i. Point of inflexion **[B1]**

ii. Differentiates ($\frac{dy}{dx} = x^2 - 6x + 9$) **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$, inverts and changes sign $(= -1)$ **[M1]**

$y = -x + \frac{32}{3}$ or equivalent (must be in this form) **[A1]**

[Allow anything rounding to 10.7 for $\frac{32}{3}$]

iii. $\frac{32}{3}$ **[B1]**

iii. Finds the area of the triangular section ($\frac{1}{2} \times \frac{26}{3} \times \frac{26}{3}$ or integrating their equation of line l from $x = 2$ to $x =$ their value of a) $[= \frac{338}{9}]$ **[M1*]**

Integrates the equation of the curve (presence of any $x -$ term with an index raised by 1 is sufficient for this mark $(\frac{x^4}{12} - x^3 + \frac{9x^2}{2})$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round $(\frac{34}{3} - 0)$ **[M1]**

Adds their areas $(\frac{338}{9} + \frac{34}{3})$ **[M1]**

$= \frac{440}{9}$ or equivalent, or anything rounding to 48.9 **[A1]**

Question B4

a) i.

Mid-value (x)	**Frequency** (f)	$x \times f$	**Interval width**	**Frequency density**
1	14	14	2	7.0
3	10	30	2	5.0
6	16	96	4	4.0
10	6	60	4	1.5
16	4	64	8	0.5

$$\sum(x \times f) = 264$$

Completes $x \times f$ column **[M1]**

Their $\sum(x \times f) \div 50$ **[M1]**

$= 5.28$ **[A1]**

ii. $0 < t \leq 2$ (Accept $0 - 2$) **[B1]**

iii. Correct frequency densities **[B1]**

(A sketch of the histogram is on page 15)

Correctly labelled and scaled horizontal axis **[B1]**

Boundaries of columns in correct places **[B1]**

Heights of columns correct to give appropriate areas. **[B1ft]**

Allow follow through for their frequency densities provided a reasonable histogram emerges.

(An accurate histogram which is not drawn on graph paper scores a maximum of 3 marks out of 4.)

iv. $\frac{1}{2} \times 10 + \frac{1}{4} \times 16$ or similar **[M1]**

$= 9$ **[A1]**

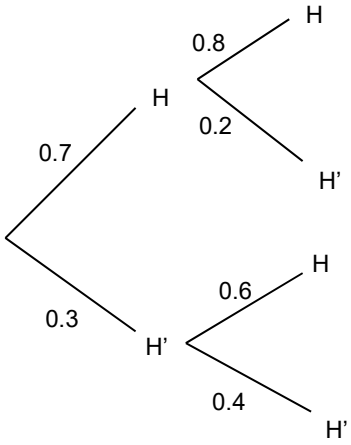
Part b) is on the next page.

Question B4 - (continued)

b) i.

FIRST SHOT SECOND SHOT

H denotes the event of a hit



(B1) first set of branches correct; **(B1)** second set of branches correct **[B2]**

ii. 0.7×0.8; their 0.3×0.6 **[M1]**

Adds **[M1]**

0.74 **[A1]**

iii. $\frac{\text{their } 0.3 \times 0.6}{\text{their part ii}}$ **[M1]**

$= \frac{9}{37}$ or anything rounding to 0.243 (Allow follow through provided the answer is between 0 and 1.) **[A1ft]**

c) i. No, because $p(C \cap D) \neq 0$ or 'C and D can both happen at the same time', or any other valid explanation. **[B1]**

ii.

Yes **(B1*)** because $p(C) \times p(D) = 0.45 \times 0.8 = 0.36 = p(C \cap D)$

or $p(C|D) = \frac{0.36}{0.8} = 0.45 = p(C)$ or $p(D|C) = \frac{0.36}{0.45} = 0.8 = p(D)$ **(B1)** **[B2]**

*Only allow this mark if a reason (even a wrong one) follows.

Question B5

a) i.

x^2	xy
0	0
4	-22
9	12
25	10
16	12

(B1) for each column **[B2]**

ii. $[\sum x = 10;\ \ \sum y = 30;\ \ \bar{x} = 2;\ \ \bar{y} = 6\ \sum x^2 = 54;\ \ \sum xy = 12\ \ ]$

$s_x^2 = \frac{\text{their } \sum x^2}{5} -$ their $\bar{x}$ squared $(= 6.8)$ **[M1*]**

$s_{xy} = \frac{\text{their } \sum xy}{5} -$ their $\bar{x} \times$ their $\bar{y}$ $(= -9.6)$ **[M1*]**

$y -$ their $\bar{y} =$ $\frac{\text{their } s_{xy}}{s_x^2}$ $(x -$ their $\bar{x})$ **[M1*]**

$y - 6 = -1.41\ (x - 2)$ or $y = -1.41x + 8.82$ **[A1ft]**
[Allow anything rounding to -1.4 and 8.8]

Allow follow through.

[If the correct equation appears with no working, give 1 mark out of 4]

iii. Substitutes $x = -4$ into their equation **[M1]**

= anything rounding to 14.5 (Allow follow through) **[A1ft]**

iv. Not reliable/not very reliable because -4 is outside the range of readings on which the equation was formed (or similar words). **[B1]**

b) i. $19.8 \pm \frac{0.4 \times 1.96}{\sqrt{49}}$ **[M1]**

Anything rounding to 19.7 **(A1)** anything rounding to 19.9 **(A1)** **[A2]**

ii. (Probably) false as 20 is outside the confidence interval (but only just) **[B1ft]**
[Allow follow through on their confidence interval]

Part c) is on the next page.

Question B5 - (continued)

c) i. $p = 0.1$ **[B1]**

ii. $[(0 \times \text{their } p)] + (1 \times 0.25) + (4 \times 0.35) + (6 \times 0.18) + (q \times 0.12)$ **[M1]**

$= 3.93$

$q = 10$ **[A1]**

iii. $\mathrm{E}(X^2) = [(0 \times \text{their } p)] + (1 \times 0.25) + (16 \times 0.35) + (36 \times 0.18)$
$+ (\text{their } q^2 \times 0.12) \quad [= 24.33]$ **[M1]**

$\mathrm{Var}(X) = \text{their } \mathrm{E}(X^2) - 3.93^2$ **[M1]**

Anything rounding to 8.9 **[A1]**

iv. $\mathrm{E}(Y) = -8.72$; $\mathrm{Var}(Y) =$ anything rounding to 142 [Allow follow through on their $\mathrm{Var}(X)$]. **[B1]**

Question B6

a) i. $6x + 3x\frac{dy}{dx} + 3y - 2y\frac{dy}{dx} = 0$

Uses Product Rule (sight of $3x\frac{dy}{dx} + 3y$ is sufficient for this mark) **[M1*]**

Correct implicit differentiation (sight of $x\frac{dy}{dx}$ or $y\frac{dy}{dx}$ is sufficient) **[M1*]**

Gathers $\frac{dy}{dx}$ terms on to one side and factorises [this mark is available only if there are at least two $\frac{dy}{dx}$ terms present] **[M1]**

$\frac{dy}{dx} = \frac{-6x - 3y}{3x - 2y}$ or equivalent **[A1]**

ii. $-6x - 3y = 0$ (this must be seen) so $y = -2x$ **[M1*]**

iii. Substitutes $y = -2x$ into original expression **[M1]**

Reaches at least one value for x $(= \pm 2)$ **[M1]**

Coordinates are $(2, -4)$; $(-2, 4)$. **[A1]**

b) Uses the Quotient Rule $\left[\left(\frac{dy}{dx} =\right) \frac{\cos x(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}\right]$ **[M1*]**
(The Quotient Rule must be used)

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ (This stage must be seen) **[M1*]**

$= \frac{1}{\cos^2 x}$ (Both M marks scored and no errors seen) **[A1]**

c) i. $3x^2 + 10x + 9 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$ **[M1]**

$A = 2$; $B = 1$; $C = -1$ **(A1)** for each **[A3]**

ii. Uses previous result $\left[\int_0^1 \left(\frac{2}{x+1} + \frac{1}{x+2} - \frac{1}{(x+2)^2}\right) dx\right]$ **[M1*]**

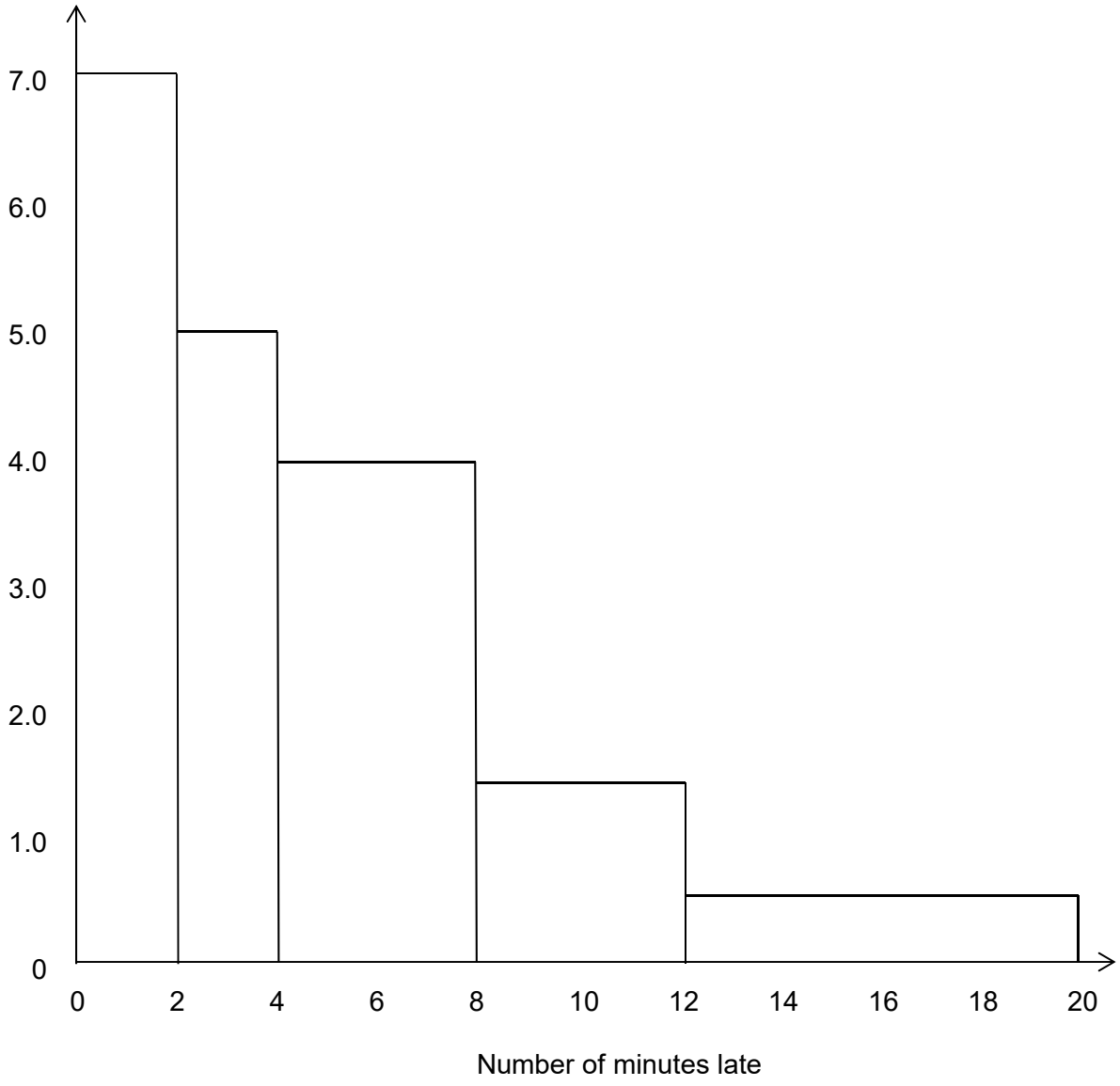
Integrates (sight of a log term or reciprocal $(x+2)$ is sufficient for this mark $\left[2\ln(x+1) + \ln(x+2) + \frac{1}{x+2)}\right]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

Uses any one of the logarithm laws (power, addition or subtraction) **[M1]**

$= \ln 6 - \frac{1}{6}$ **[A1]**

Histogram for question B4



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