NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination
2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A7. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Gradient of BC is $\frac{7}{14}$ or equivalent **[M1]**

Finds the equation of BC $(y = \frac{1}{2}x + 6)$ **[M1]**

Substitutes $x = -20$ into their equation and finds a value for k. **[M1]**

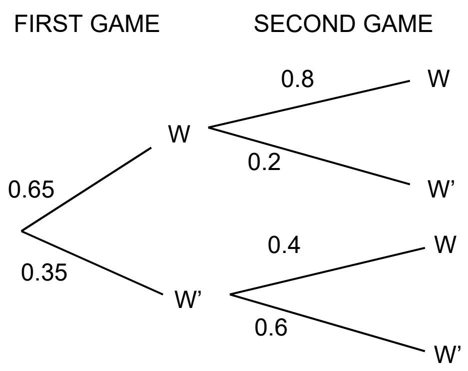
$k = -4$ **[A1]**

or sets the gradient of BC equal to $\frac{2 - k}{-8 - -20}$ or $\frac{9 - k}{6 - -20}$ for the second M1

Rearranges and finds a value for k for the third M1

Question A2

a)



First set of branches correct **[B1]**

Second set of branches correct **[B1]**

Where W denotes the event 'wins a game'

b) 0.65 × their 0.2, their 0.35 × 0.4 **[M1]**

Adds **[M1]**

= 0.27 **[A1]**

Question A3

Factorises or uses formula $[(2x+5)(x-4)=0 \text{ or } x=\frac{3 \pm \sqrt{[(-3)^2 - 4 \times 2 \times -20]}}{2 \times 2}]$ **[M1]**

Finds two critical values $(-\frac{5}{2},\ 4)$ **[M1]**

$x \geq -\frac{5}{2}$ **(A1)*** $x \leq 4$ **(A1)*** or $-\frac{5}{2} \leq x \leq 4$ **(A1)** for each end. **[A2]**

*Please note: if this version of the answers is quoted, the two ranges can be separated by a space, a comma or the word ‘and’. The final mark is lost if the word ‘or’ is seen.

Question A4

${}^6C_4 \times p^2 \times (-3)^4$ (Allow the presence of x and ${}^4C_6$ for ${}^6C_4$) **[M1]**

Sets equal to 135 and reaches $p^2 = \cdots (\frac{1}{9})$ (There must now be no x present) **[M1]**

$p = \pm\frac{1}{3}$ **[A1]**

Question A5

$\log_8[(x-4)(x-5)] = \log_8 2$

Correct use of the addition law of logs **[M1*]**

Adapts RHS and removes logs at the right time **[M1*]**

Forms a quadratic equation $(x^2 - 9x + 18 = 0]$ **[M1]**

Factorises or uses formula $[(x-6)(x-3) \text{or } x = \frac{9 \pm \sqrt{[(-9)^2 - 4 \times 1 \times 18]}}{2 \times 1}]$ **[M1]**

$x = 6$ [if the 3 is also quoted, this mark is lost: placing it in brackets is good enough to indicate non-inclusion.] **[A1]**

Question A6

$\cos\theta = -\frac{1}{2}$ **[M1]**

$\theta = \frac{2\pi}{3},\ \frac{4\pi}{3}$ **(A1)** for each. One mark lost for extra solutions in range; ignore solutions outside range. **[A2]**

Question A7

$\frac{dy}{dx} = 3x^2 + \frac{1}{x}$, $\frac{d^2y}{dx^2} = 6x - \frac{1}{x^2}$ (Attempts to differentiate twice) **[M1*]**

Substitutes $x = 0.7$ into their $\frac{d^2y}{dx^2}$ **[M1]**

= 2.15918… (can be implied) **[A1]**

= 2.16 to 3 significant figures (Allow follow through) **[A1ft]**

Question A8

Product moment correlation coefficient $= 0.8$ **[B1]**

Equation of line of regression is $y = 0.4x + 4.8$ (B1 for 0.4 and B1 for 4.8) **[B2]**

Question A9

$0.75 = \frac{0.6}{p(A)}$ **[M1]**

$p(A) = 0.8$ **[A1]**

$0.55 = \text{their } 0.8 + p(B) - 0.6$ **[M1]**

$p(B) = 0.35$ **[A1]**

Question A10

$z = \frac{735 - 700}{20}$ $(= 1.75)$ **[M1]**

Finds Φ(their 1.75) $[= 0.9599]$ **[M1]**

Anything rounding to 4% (must be given as a percentage) **[A1]**

Question A11

Uses the Quotient Rule ($\frac{dy}{dx} =$) $\frac{3x^2(x^2 - 1) - 2x(1 + x^3)}{(x^2 - 1)^2}$ **[M1*]**

Substitutes $x = 3$ into their $\frac{dy}{dx}$ $(= \frac{3}{4})$ **[M1]**

Writes correct form of equation [$y - \frac{7}{2} = \text{their gradient}(x - 3)$] **[M1]**

$y - \frac{7}{2} = \frac{3}{4}(x - 3)$ or equivalent **[A1]**

Question A12

Uses integration by parts in right direction **[M1*]**

$8x \times \frac{1}{2}\sin 2x$ **(A1) -** $\int 8 \times \frac{1}{2}\sin 2x \; dx$ [A1 for correct first part] **[A1]**

$4x\sin 2x + 2\cos 2x + c$ (Does not need to be simplified) **[A1]**

Section B

Question B1

a) Substitutes $x = -2$ into first expression and $x = 1$ into second. [The Remainder Theorem must be used.] **[M1*]**

Sets first remainder equal to three times the second remainder and forms a quadratic equation $(3c^2 + 4c - 15 = 0)$ **[M1]**

Factorises or uses formula
$[(3c-5)(c+3) = 0$ or $c = \frac{-4 \pm\sqrt{[4^2 - 4 \times 3 \times -15]}}{2 \times 3}$ **[M1]**

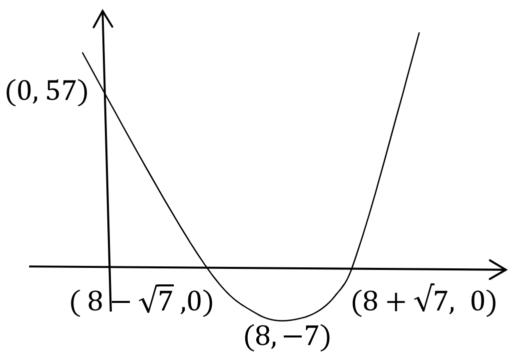
$c = -3,\ \frac{5}{3}$ or equivalent **[A1]**

b) i. $(x-8)^2$ **(B1)** -7 **(B1)** **[B2]**

ii. Writes $(x-8)^2 - 7 = 0$ and reaches $(x-8) = \pm\sqrt{\ldots}$ **[M1*]**

$x = 8 \pm \sqrt{7}$ **[A1]**

iii.



Correct shape **[B1]**

(0, 57) **[B1]**

(8, -7) **[B1]**

$(8 - \sqrt{7}, 0)$; $(8 + \sqrt{7}, 0)$ **[B1]**

c) i. $(a = 6,\ d = 4)$ $70 = 6 + (n-1) \times 4$ **[M1]**

$n = 17$ **[A1]**

ii. $S_{31} = \frac{31}{2}[2 \times 6 + (31-1) \times 4]$ **[M1]**

Calculates in correct order **[M1]**

= 2046 **[A1]**

d) $\frac{a[(-2)^{12} - 1]}{-2 - 1} = -8190$ **[M1]**

Solves **[M1]**
$a = 6$ **[A1]**

Question B2

a) $4m^{-3} = 2048$ Adds or subtracts indices correctly at any time **[M1]**

Reaches $m^{-3} = (512)$ or $m^3 = (\frac{1}{512})$ **[M1]**

$m = \frac{1}{8}$ **[A1]**

b) i. Rearranges and reaches $e^{2k} = \cdots (\frac{216}{24})$ **[M1*]**

Takes logs correctly and reaches $2k = \cdots (\ln 9)$ **[M1*]**

$k = \frac{1}{2}\ln 9$ or $\ln 9^{½}$ (this step must be seen) $= \ln 3$ **[A1]**

ii. Substitutes $q = 4$ into formula and finds a value for p **[M1]**

$p = 1908$ **[A1]**

iii. $\frac{dp}{dq} = 24 \times \ln 3 \times e^{q \times \ln 3}$ **[M1]**

Substitutes $q = 2.5$ into their $\frac{dp}{dq}$ **[M1]**

Anything rounding to 411 **[A1]**

iv. $24e^{q \times \ln 3}$ is always greater than 0, (so $p > -36$), so p never reaches -40 (or a similar explanation which shows the necessary understanding). **[B1]**

c) i. Uses cosine formula $(1300 = 34^2 + 36^2 - 2 \times 34 \times 36 \times \cos\theta)$ **[M1]**

Calculates in correct order **[M1]**

$\cos\theta = \frac{8}{17}$ **[A1]**

ii. *Please note: this is a 'show that' question so all working must be seen*.

Uses $\cos^2\theta + \sin^2\theta = 1$, a right-angled triangle or any other valid method **[M1*]**

Reaches $\sin\theta = \frac{15}{17}$ (M mark scored and no errors seen) **[A1]**

iii. $\frac{1}{2} \times 34 \times 36 \times \frac{15}{17}$ **[M1]**

= 540 (cm^2) **[A1]**

iv. $\frac{17}{18}$ **[B1]**

Question B3

a) i. Integrates $(-\cos x)$, substitutes limits into their integrated expression and subtracts the right way round. **[M1*]**

= 0 **[A1]**

ii. Equal areas above and below $x-$ axis or similar words, but 'there is zero area' scores B0 **[B1]**

b) i. Attempts to differentiate (sight of x^2 in the first term, x in the second, or -24 in the third is sufficient for this mark $(6x^2 - 18x - 24)$ **[M1*]**

Sets equal to 0 (can be implied) **[M1]**

Factorises or uses formula
$[6(x-4)(x+1) = 0$ or $x = \frac{18 \pm\sqrt{[(-18)^2 - 4\times 6 \times -24]}}{2\times 6}$ **[M1]**

Substitutes their x –values into original equation and finds at least one $y-$ value. **[M1]**

$(-1, 17)$; $(4, -108)$ **[A1]**

ii. Finds $\frac{d^2y}{dx^2}$ $(12x - 18)$ **[M1*]**

Substitutes their $x-$ values into their $\frac{d^2y}{dx^2}$ **[M1*]**

Shows $\frac{d^2y}{dx^2}$ is negative when $x = -1$ and positive when $x = 4$ **[M1*]**

Maximum at $(-1,\ 17)$ Minimum at $(4, -108)$ **[A1ft]**

or Takes a numerical value below -1 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Takes a numerical value between -1 and 4 and shows $\frac{dy}{dx} < 0$ **(M1*)**

Takes a numerical value above 4 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Maximum at $(-1,\ 17)$ Minimum at $(4, -108)$ **(A1ft)**

Part c) is on the next page.

Question B3 – (continued)

c) i. $\int_0^2 \frac{1}{4}x^4 \; dx$ and $\int_2^3 (x^2 - 9x + 18)\, dx$ (Limits must be correct) **[M1*]**

Attempts to integrate (presence of x^5 in the first case or any of x^3, x^2 or x in the second) $[\frac{1}{20}x^5$ and $\frac{1}{3}x^3 - \frac{9}{2}x^2 + 18x\,]$ **[M1*]**

Substitutes limits into both of their integrated expressions and subtracts the right way round **[M1]**

Adds their areas $(\frac{8}{5} + \frac{11}{6})$ **[M1]**

$\frac{103}{30}$ or equivalent, or anything rounding to 3.43 **[A1]**

ii. Finds $\frac{dy}{dx}$ $(2x - 9)$ **[M1*]**

Substitutes $x = 3$, inverts and changes sign $(\frac{1}{3})$ **[M1]**

$y = \frac{1}{3}(x - 3)$ or equivalent **[A1]**

Question B4

a) i.

Mid-value (x)	**Frequency (f)**	$x \times f$	**Cum. Freq.**
5	3	15	3
15	7	105	10
25	12	300	22
35	22	770	44
45	19	855	63
55	12	660	75
65	5	325	80

$\sum x \times f = 3030$

Works out $x \times f$ values **[M1]**

Divides their $\sum x \times f$ by 80 **[M1]**

$37\frac{7}{8}$ or anything rounding to 37.9 **[A1]**

ii. Finds cumulative frequencies **[M1]**

Plots correct curve (a sketch is on page 15). 1 mark lost for each omitted/incorrect plot; 1 mark lost for each point missed by the curve by at least 1 mm (but allow ft for any incorrect plots); 1 mark lost if either axis not labelled correctly.
(<u>Please note</u>: a maximum total of 3 marks can be lost i.e. there are no negative scores. If the candidate plots the mid-values instead of the upper values in each interval, this will score A0.)

If graph paper is not used, award 1 mark out of the A3 if a reasonable curve is drawn. If a cumulative frequency polygon is drawn, award up to 2 marks out of the A3. **[A3]**

iii. Correctly reads off their median (should be around 38) **[M1]**

Reads off their lower quartile (should be around 28) and their upper quartile (should be around 48) **[M1]**

Subtracts and obtains interquartile range (should be around 20) **[M1]**

iv. No **(B1*)** valid reasons could be ‘mean and median are close to each other’; ‘there is no obvious “bunching” of data at the lower or upper end of the range’; ‘the cumulative frequency curve indicates a (reasonably) symmetric distribution’, etc. **(B1)**

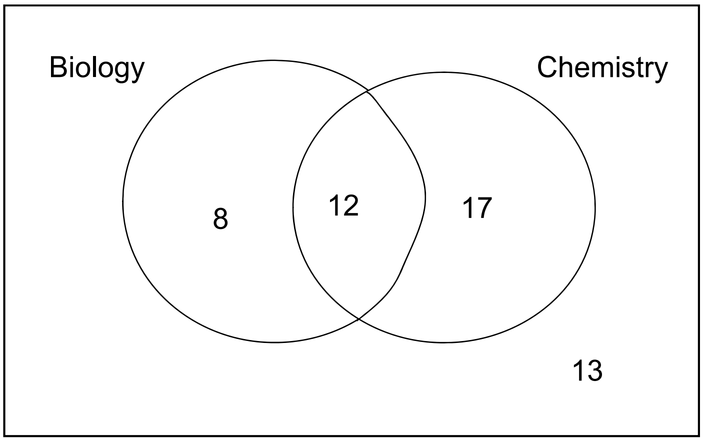
*Only award this mark if a reason (even a wrong one) follows. **[B2]**

Part b) is on the next page.

Question B4 - (continued)

b) ii.

E



B1 – any one correct entry; **B2** any two correct entries
B3 – all entries correct and rectangle drawn. Condone missing E. **[B3]**

iii. $p(B') = \frac{30}{50}$; $p(B \cap C') = \frac{8}{50}$; $p(B' \cup C') = \frac{38}{50}$

B1 for each. Accept percentages, decimals and equivalent fractions. Allow follow through from their Venn diagram. **[B3ft]**

iv. Finds $p(B \cap C)\ (= \frac{12}{50})$ and $p(B) \times p(C)\ (= \frac{29}{125})$ **[M1*]**

As $p(B \cap C) \neq p(B) \times p(C)$ the events are not independent (or similar conclusion). Allow follow through from their Venn diagram. **[A1ft]**

Question B5

a) Amount = £2388 **[M1]**

$2000(1+\frac{r}{100})^6 = 2388$ **[M1]**

Solves by using logs or taking the sixth root, and reaches a value for r **[M1]**

r = anything rounding to $3(\%)$ **[A1]**

b) i. $(\bar{x} = 39;\ \bar{y} = 16)$ s_x = anything rounding to 11.5 **[B1]**

s_y = anything rounding to 3.29 **[B1]**

$s_{xy} = 36.6$ **[B1]**

ii. Anything rounding to 0.96 or 0.97. Allow follow through **[B1ft]**

iii. Yes **(B1*)** because the correlation is a (very) strong positive one (or similar comment) **(B1)**. Allow follow through for their PMCC **[B2ft]**

*Only award this mark if a reason (even a wrong one) follows.

c) i. Anything rounding to 0.414 **[B1]**

ii. $1 - p(10 \text{ or less}) \quad (1 - 0.7507)$ **[M1]**

Anything rounding to 0.249 **[A1]**

iii. Exactly 9 are aged 50 or above **[M1]**

${}^{20}C_9 \times 0.45^9 \times 0.55^{11}$ or $p(\leq 9) - p(\leq 8)$ **[M1]**

Anything rounding to 0.177 **[A1]**

d) i. $E(X^2) = (0 \times 0.15) + 1 \times 0.1 + 4 \times 0.25 + 16 \times 0.3 + 49 \times 0.2$ $(= 15.7)$ **[M1]**

$\text{Var}(X) = \text{their E}(X^2) - 3.2^2$ **[M1]**

$= 5.46$ **[A1]**

ii. $\text{E}(Y) = 8.6;\quad \text{Var}(Y)$ = anything rounding to 49.1 (Allow follow through) **[B1ft]**

Question B6

a) i. $4\cos^3 x$ or $-\sin x$ seen **[M1]**

$\left(\frac{dy}{dx} =\right) -4\cos^3 x \sin x$ **[A1]**

ii. Uses Product Rule **[M1]**

$\left(\frac{dy}{dx} =\right) -4\cos^3 x \sin x \tan x + \frac{\cos^4 x}{\cos^2 x}$ or $-4\cos^2 x \sin^2 x + \cos^2 x$ **[A1]**

or any equivalent answer. There is no need to simplify.

iii. $3 + 10x + 3y^2 \frac{dy}{dx} - 8\frac{dy}{dx} = 0$

Uses implicit differentiation (sight of $y^2 \frac{dy}{dx}$ or $8\frac{dy}{dx}$ is sufficient) **[M1]**

Takes $\frac{dy}{dx}$ terms on to one side and factorises (this mark can be given only if the number of $\frac{dy}{dx}$ terms is two or more) **[M1]**

$\left(\frac{dy}{dx} =\right) \frac{-3 - 10x}{3y^2 - 8}$ **[A1]**

b) i. $x^2 - 3x - 2 = Ax(x-2) + B(x-2) + Cx^2$ **[M1]**

$A = 2;$ $B = 1;$ $C = -1$ **(A1)** for each **[A3]**

ii. Uses previous result $(\int_3^4 \frac{2}{x} + \frac{1}{x^2} - \frac{1}{x-2}\ dx$ **[M1*]**

Integrates (sight of a log term or $\frac{1}{x}$ is sufficient for this mark) **[M1*]**

$[\,2\ln x - \frac{1}{x} - \ln(x-2)]$

Applies a correct log power law or correct subtraction law at any stage **[M1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$\ln\left(\frac{8}{9}\right) + \frac{1}{12}$ **[A1]**

Part c) is on the next page.

Question B6 – (continued)

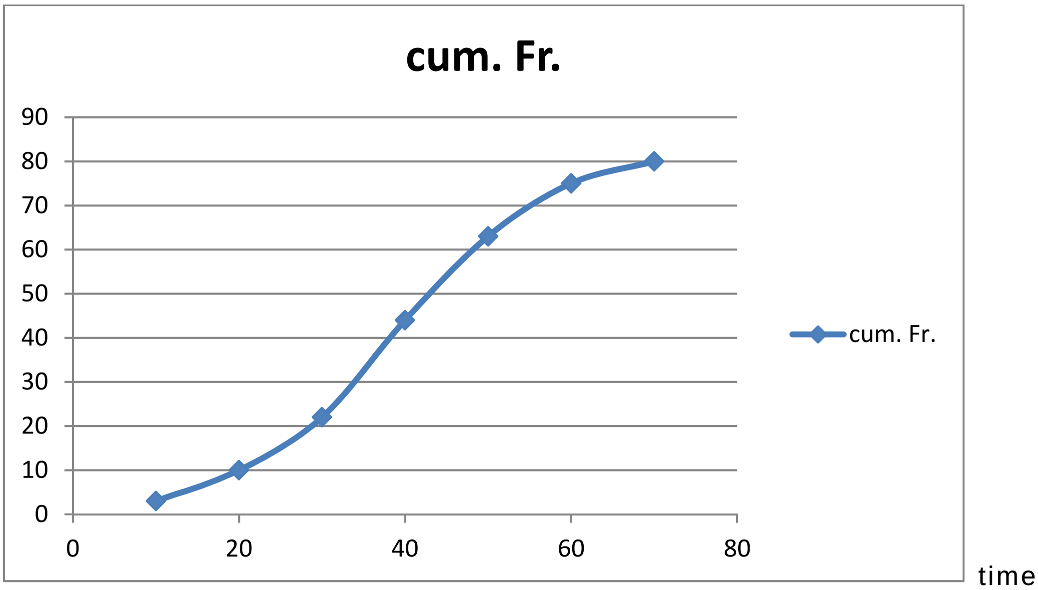
c) $du = 2e^{2x}\,dx$ or equivalent **[M1*]**

Writes integral in terms of u ($\int \frac{1}{2}\, u^5 \;du$) **[M1*]**

Integrates [$\frac{1}{12}u^6\ (+c)$] and changes back to terms in x **[M1]**

$\frac{(3 + e^{2x})^6}{12} + c$ **[A1]**

Sketch for Question B4 a) ii.



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