THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB002 Mathematics Business
Examination
2017-18

MARK SCHEME

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Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

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Section A

Question A1

Uses any method to find one unknown **[M1]**

Uses any method to find second unknown **[M1]**

$c = -\frac{1}{4}$ or equivalent **[A1]**

$d = 3$ **[A1]**

Question A2

p(both red) $= \frac{3}{5} \times \frac{2}{4}$ **[M1]**

p(both green) $= \frac{2}{5} \times \frac{1}{4}$ **[M1]**

Adds their probabilities **[M1]**

$= \frac{2}{5}$ or equivalent **[A1]**

Question A3

Please note: the Remainder Theorem must be used in this question.

Substitutes $x = 4$ into expression and sets equal to $3k$ **[M1*]**

Solves **[M1]**

$k = -29$ (Just quoting the answer with no working scores 0 marks) **[A1]**

Question A4

$(2x)^4 + {}^4C_1\,(2x)^3\,(-3) + {}^4C_2\,(2x)^2\,(-3)^2 + {}^4C_3\,(2x)(-3)^3 + (-3)^4$

(B1) Any two unsimplified correct; **(B2)** all unsimplified correct

[For the first two marks, allow ${}^xC_y$ for ${}^yC_x$] **[B4]**

$= 16x^4 - 96x^3 + 216x^2 - 216x + 81$

(B1) Any two simplified correct; **(B2)** all simplified correct

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Question A5

Recognises ‘hidden’ quadratic equation **[M1]**

Factorises or uses formula
$[\ (3^x - 1)(3^x - 9) = 0 \text{ or } 3^x = \frac{10\ \pm\ \sqrt{[(-10)^2 - 4 \times 1 \times 9]}}{2\ \times\ 1}\ ]$ **[M1]**

Finds values for 3^x $(1,\ 9)$ and goes on to find at least one value for x. **[M1]**

$x = 0,\ 2$ **[A1]**

Question A6

$\cos\theta = -\frac{4}{5}$ **[M1]**

$\theta = 2.49809 \ldots$ (can be implied) **[A1]**

$= 2.50$ to 3 significant figures (Allow follow through) **[A1ft]**

Special case: If 143 is seen (answer in degrees) award 2 marks out of 3

Question A7

Breaks up integrand $(\frac{3x^3}{x} - \frac{2}{x})$ and attempts to integrate (sight of x^3 or $\ln x$ is sufficient for this mark). **[M1]**

$x^3 - 2\ln x + c$ **(A1)** for one correct part; **(A2)** all correct and $+\,c$ **[A2]**

Question A8

Mean $= 3$; median $= 2.5$; range $= 9$ **(B1)** for each **[B3]**

Question A9

$\frac{p\ +\ 5\ +\ 4p\ -\ 1\ +\ 3p\ +\ 5}{3} = p^2 + 2$ and forms quadratic equation $(3p^2 - 8p - 3 = 0)$ **[M1]**

Factorises or uses formula $[(3p + 1)(p - 3) = 0 \text{ or } p = \frac{8\ \pm\ \sqrt{[(-8)^2\ -\ 4\ \times\ 3\ \times\ (-3)]}}{2\ \times\ 3}]$ **[M1]**

$p = 3$ (ignore any reference to $-\frac{1}{3}$) **[A1]**

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Question A10

$\mathrm{E}(X) = -1 \times 0.1\ (+0 \times 0.15) + 1 \times 0.2 + 2 \times 0.25 + 3 \times 0.3$ **[M1]**

$= 1.5$ **[A1]**

$\mathrm{E}(X^2) = 1 \times 0.1\ (+0 \times 0.15) + 1 \times 0.2 + 4 \times 0.25 + 9 \times 0.3\ (= 4)$ **[M1]**

$\mathrm{Var}(X) = \text{their } \mathrm{E}(X^2) - \text{their } [\mathrm{E}(X)]^2$ and takes square root **[M1]**

$= \sqrt{1.75}$ or anything rounding to 1.32 **[A1]**

Question A11

Uses the Quotient Rule or Product Rule $[\frac{dy}{dx} = \frac{2x(x+1) - (x^2+3)}{(x+1)^2}$ **[M1*]**
or $y = (x^2+3)(x+1)^{-1}$ so $\frac{dy}{dx} = (2x)(x+1)^{-1} - (x^2+3)(x+1)^{-2}\ ]$

Sets equal to 0 and forms a quadratic equation $(x^2 + 2x - 3 = 0)$ **[M1]**

Factorises or uses formula $[(x+3)(x-1) = 0 \text{ or } x = \frac{-2 \pm \sqrt{[2^2 - 4 \times 1 \times (-3)]}}{2 \times 1}]$ **[M1]**

Finds two values of x, substitutes at least one of these into the original equation and finds a value for y. **[M1]**

$(-3, -6)$; $(1,\ 2)$ **[A1]**

Question A12

$du = 3x^2\,dx$ or equivalent **[M1*]**

Writes integral in terms of u $(\int \frac{1}{3} \times u^{½}\ du)$ and attempts to integrate
(the limits do not have to be changed for this mark) **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. If the limits have not been changed, then expression must be converted back into terms in x. **[M1]**

$= \frac{52}{9}$ or anything rounding to 5.78 **[A1]**

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Section B

Question B1

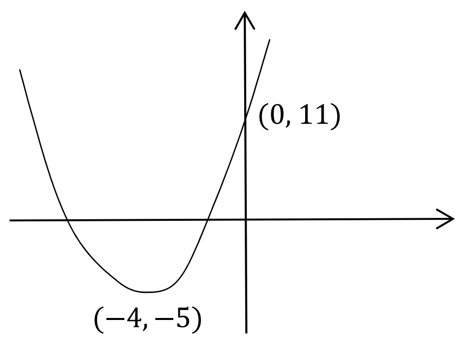
a) Finds gradient $\left(-\frac{4}{3}\right)$, inverts and changes sign $\left(\frac{3}{4}\right)$ **[M1]**

Writes correct form of equation with their gradient **[M1]**

$y - 12 = \frac{3}{4}(x - 8)$ or equivalent **[A1]**

b) i. $(x + 4)^2$ **(B1)** -5 **(B1)** **[B2]**

ii.



Correct shape **[B1]**

$(-4, -5)$ shown **[B1]**

$(0, 11)$ shown **[B1]**

c) i. $(a = 8,\ r = 1.5)$ $\quad \mathrm{U}_7 = 8 \times (\text{their } 1.5)^6$ **[M1]**

$91\frac{1}{8}$ or equivalent **[A1]**

ii. $\frac{8[(\text{their } 1.5)^n - 1]}{\text{their } 1.5 - 1} = 1000000$ **[M1]**

Rearranges and uses logs correctly **[M1]**

$n = 27.235 \ldots$ **[A1]**

28 terms needed. **[A1]**

iii. The series does not converge or similar explanation. The value of r does not lie between -1 and 1 is good enough. **[B1]**

Part d) is on the next page.

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Question B1 – (continued)

d) i.

$$
\begin{array}{r|l}
 & 2x^2 + 9x \;\; + \;\; 9 \\
\hline
x - 7 & 2x^3 - 5x^2 - 54x - 63 \\
 & \underline{2x^3 - 14x^2} \\
 & \qquad 9x^2 - 54x \\
 & \qquad \underline{9x^2 - 63x} \\
 & \qquad\qquad 9x - 63 \\
 & \qquad\qquad \underline{9x - 63} \\
 & \qquad\qquad \underline{\ldots\ldots\ldots}
\end{array}
$$

Correct first division **[M1]**

Any correct subsequent division **[M1]**

Correct quotient **[A1]**

ii. $(x-7)(2x+3)(x+3)$ **[B1]**

iii. $x = -3, \ -\frac{3}{2}, \ 7$ **[B1ft]**

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Question B2

a) i. Because $C = 30$ when $t = 0$ which makes $A = 30$ or any similar explanation but be on the lenient side here. **[B1]**

ii. $30e^{-0.03 \times 4}$ **[M1]**

$26.6\ (\%)$ **[A1]**

iii. Substitutes $C = 15$, rearranges and reaches $e^{-0.03t} = \cdots \quad (\frac{15}{30})$ **[M1]**

Takes logs and reaches $-0.03\,t = \cdots \quad (\ln\frac{15}{30})$ **[M1]**

$(t = 23.1)$ so harvest is ready after 23 days. If the answer is given as 24 days, then allow as long as $t = 23.1$ is seen. **[A1]**

b) $\log_2\left[\frac{(x+6)^2}{(x+6)(x-1)}\right] = \log_2 8$ Uses the log power law correctly **[M1*]**

Uses the log subtraction law correctly **[M1*]**

Adapts the RHS and removes logs at the correct time **[M1*]**

Solves the equation **[M1]**

$x = 2$ **[A1]**

Please note: if the cancellation is missed, the last M mark can be given for forming a quadratic equation $7(x^2 + 4x - 12 = 0)$ and solving it. The A mark can be given for $x = 2$ but it is lost if the -6 is also quoted. Putting it in brackets is sufficient evidence that it is excluded.

c) Uses $\cos^2\theta + \sin^2\theta = 1$, uses a right-angled triangle or uses any other valid method **[M1*]**

$\cos\theta = \frac{35}{37}$ **[A1]**

d) i. Uses the cosine formula $(AD^2 = 10^2 + 9^2 - 2 \times 10 \times 9 \times \cos 60)$ **[M1]**

Calculates in the correct order **[M1]**

$AD = \sqrt{91}$ or anything rounding to 9.54 (cm) **[A1]**

ii. Uses sine formula $(\frac{\sin ADC}{10} = \frac{\sin 60}{\text{their } AD})$ **[M1]**

= anything rounding to 65.2 (degrees) **[A1]**

iii. $\frac{1}{2} \times 10 \times 9 \times \sin 60 = \frac{1}{2} \times 8 \times 10 \times \sin BAC$ **[M1]**

Anything rounding to 77 (degrees) **[A1]**

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Question B3

a) i. $294\pi = 2\pi rh + 2\pi r^2$ and rearranges **[M1]**

$h = \frac{294\pi - 2\pi r^2}{2\pi r}$ or equivalent (such as $\frac{147}{r} - r$) **[A1]**

Please note: this is a 'show that' question so all working must be seen.

ii. $V = \pi r^2(\text{their } h)$ **[M1*]**

At least one correct intermediate line of working + **[M1*]**

$V = 147\pi r - \pi r^3$ (Both M marks scored and no errors seen) **[A1]**

+ Special case: if the candidate has used $h = \frac{147 - r^2}{r}$, the intermediate line of working does not need to be shown, but the mark can still be given as some additional work has been done, even if it was in part i.

iii. $\frac{dV}{dr} = 147\pi - 3\pi r^2$ Attempts to differentiate (Presence of the π term or r^2 is sufficient for this mark.) **[M1*]**

Sets their $\frac{dV}{dr}$ equal to 0 (can be implied) **[M1]**

Reaches $r^2 = \cdots (49)$ **[M1]**

$r = 7$ **[A1]**

iv. $\frac{d^2V}{dr^2} = -6\pi r$ Attempts to differentiate a second time. Sight of r is sufficient for this mark. **[M1*]**

Correct answer **[A1]**

This is negative (when $r = 7$), so there is a maximum (reason and conclusion). Allow follow through provided their $\frac{d^2V}{dr^2}$ gives a negative value. **[A1ft]**

or takes a numerical value of r between 0 and 7 and shows $\frac{dV}{dr} > 0$ **(M1*)**

takes a numerical value of r above 7 and shows $\frac{dV}{dr} < 0$ **(M1*)**

thus there is as maximum (when $r = 7$) **(A1ft)**
[Allow follow through on their $\frac{dV}{dr}$ provided it gives a maximum]

Part b) is on the next page.

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Question B3 – (continued)

b) i. Differentiates ($\frac{dy}{dx} = 2 - 2x$) and substitutes $x = 2 \quad (-2)$ **[M1]**

$y - 3 = \text{their gradient}(x-2)$ or $y = \text{their gradient } \times x + c$ and attempts to find a value for c. **[M1]**

$y = -2x + 7$ (Must be in this form) **[A1]**

ii. Area under line = $\frac{1}{2}(\text{ their } 7 + 3) \times 2 \quad (= 10)$ or $\int_0^2 (-2x + 7) \quad dx$ **[M1*]**

Area under curve = $\int_0^2 (3 + 2x - x^2) \quad dx$ and attempts to integrate **[M1*]**

$[\, 3x + x^2 - \frac{x^3}{3}\,]$ (Presence of $x,\ x^2$ or x^3 is sufficient for this mark)

Substitutes limits into their integrated expression and subtracts the right way round $(\frac{22}{3})$ **[M1]**

Subtracts one area from the other **[M1]**

$= \frac{8}{3}$ or anything rounding to 2.67 **[A1]**

<u>Alternative solution</u>

Subtracts equation of curve from equation of line **(M1)**

Area = $\int_0^2 (4 - 4x + x^2) \quad dx$ **(M1*)**

Attempts to integrate **(M1*)**

Substitutes limits into their integrated expression and subtracts the right way round. **(M1)**

$= \frac{8}{3}$ or anything rounding to 2.67 **(A1)**

<u>Please note</u>: candidates who obtain the wrong equation in part i can still score all of the M marks in part ii, but not the A mark.

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Question B4

a)

Mid-value (x)	**Frequency** (f)	$x \times f$	$x^2 \times f$
2	2	4	8
7	6	42	294
12	8	96	1152
17	7	119	2023
22	5	110	2420

i. Finds $\sum x \times f \ (= 371)$ **[M1]**

Divides by 28 **[M1]**

$= 13\frac{1}{4}$ (Allow 13.3) **[A1]**

Finds $\sum x^2 \times f \ (= 5897)$ **[M1]**

Divides their 5897 by 28, subtracts square of their mean and takes square root **[M1]**

$=$ anything rounding 5.8 or 5.9 **[A1]**

ii. Mid-values used (or words to this effect) **[B1]**

iii. 15 - 19 **[B1]**

iv. No **(B1*)** with a sensible reason **(B1)** such as 'because a more popular telephone may come on to the market', 'the economic situation may change', 'the amount of data is small so predictions cannot be made on these readings', etc.'
*This mark can only be awarded if a reason (even a wrong one) follows. **[B2]**

Part b) is on the next page.

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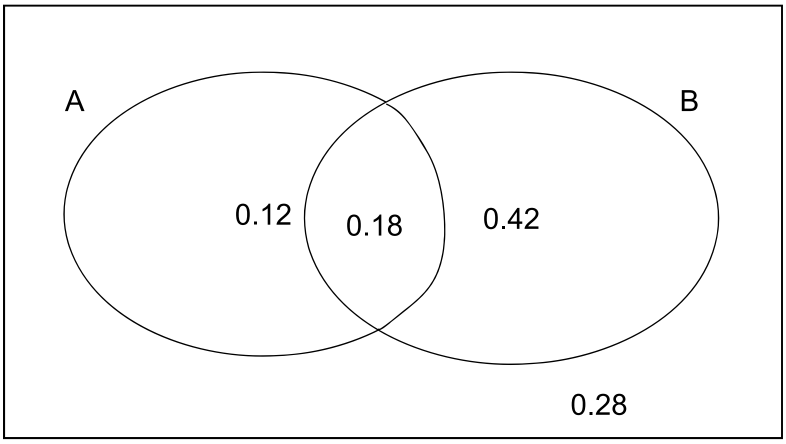
Question B4 – (continued)

b) i. *Please note: this is a 'show that' question so all working must be seen*.

$0.72 = 0.3 + p(B) - 0.18$ (or any other valid working) **[M1*]**

$p(B) = 0.6$ (M mark scored and no errors seen) **[A1]**

ii. E



Any correct entry **(B1)**; any two correct entries **(B2)**; All entries correct and contained in rectangle **(B3)** [condone missing E] **[B3]**

iii. $p(A' \cup B) = 0.88$; $p(A \cap B') = 0.12$; $p(A|B) = 0.3$

(B1) for each. Allow equivalent percentages and fractions **[B3]**

iv. $p(A \cap B) = 0.18$; $p(A) \times p(B) = 0.3 \times 0.6 = 0.18$ **[M1*]**

or $p(A|B) = 0.3 = p(A)$ **or** $p(B|A) = 0.6 = p(B)$

Thus events A and B are independent **[A1]**

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Question B5

a) i. $1271 \div \frac{102.5}{100}$ or similar working **[M1]**

$£1240$ **[A1]**

ii. $1271\,(1 + \frac{2.5}{100})^4$ or their $1240\,(1 + \frac{2.5}{100})^5$ **[M1]**

Anything rounding to $£1403$ **[A1]**

iii. Anything rounding to $£163$ **[B1ft]**

b) i. ($\bar{x}$ = 44; $\bar{y}$ = 55) $s_x^2 = 419\frac{2}{3}$ or anything rounding to 420 **[B1]**

$s_{xy} = 61\frac{2}{3}$ or anything rounding to 61.7 **[B1]**

Correct formation of equation using their values **[M1]**

$y = 0.15x + 49$ (Allow anything rounding to 0.15 and 49) Allow ft **[A1ft]**

ii. Substitutes $x = 60$ into their equation **[M1]**

Anything between 57 and 58 (allow follow through on their equation) **[A1ft]**

iii. Yes, as 60 is in the range of values on which the equation was formed. **[B1]**

c) i. $\Phi(z) = 0.9$ and finds a value for z (± 1.28) **[M1]**

Performs a correct standardising **[M1]**

$x =$ anything rounding to 97.2 (grams) **[A1]**

ii. $z = \frac{117 - 110}{10}$ $(= 0.7)$ **[M1]**

Finds ϕ(their z) $(= 0.758)$ **[M1]**

Probability $= 0.242$ **[A1]**

iii. ${}^{40}C_7 \times$ their $0.242^7 \times (1 -$ their $0.242)^{33}$ **[M1]**

= anything rounding to 0.097 **[A1]**

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Question B6

a) i. $4 - x^2\frac{dy}{dx} - 2xy + 3y^2\frac{dy}{dx} = 0$

Correct use of Product Rule ($\pm x^2\frac{dy}{dx} \pm 2xy$ is sufficient evidence) **[M1*]**

Correct implicit differentiation ($x^2\frac{dy}{dx}$ or $3y^2\frac{dy}{dx}$ is sufficient evidence) **[M1*]**

Takes $\frac{dy}{dx}$ to one side and factorises (this mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{2xy - 4}{3y^2 - x^2}$ or equivalent **[A1]**

ii. $c = 2$ **[B1]**

b) Sight of $6(4x^2 - 3x + 1)^5$ or $8x - 3$ **[M1]**

$\frac{dy}{dx} = 6(8x - 3)(4x^2 - 3x + 1)^5$ or equivalent **[A1]**

c) i. $4x + 5 = A(x + 2) + B(x - 1)$ **[M1]**

$A = 3$ $B = 1$ **(A1)** for each **[A2]**

ii. Uses previous result $[\int_2^3 \frac{3}{x-1} + \frac{1}{x+2}\ dx]$ **[M1*]**

Attempts to integrate $[3\ln(x-1) + \ln(x+2)]$ Presence of a log term is sufficient evidence for this mark **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round **[M1]**

Combines the logs using any log law at any stage in the working. **[M1]**

$= \ln 10$ **[A1]**

d) Integrates by parts in the right direction **[M1*]**

$= 9x^2 \times -\frac{1}{3}e^{-3x}$ **(A1)** $- \int 18x \times -\frac{1}{3}e^{-3x}\ dx$ [A1 for first part only] **[A1]**

Integrates by parts a second time **[M1*]**

$\pm 18x \times \frac{1}{9}e^{-3x}$ **(A1)** $- \int 18 \times \frac{1}{9}e^{-3x}\ dx$ [A1 for first part only – do not worry too much about the signs] **[A1]**

$= -3x^2e^{-3x} - 2xe^{-3x} - \frac{2}{3}e^{-3x} + c$ or equivalent **[A1]**

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