

Student ID: _____



2014-2015

IFYHM001 Mathematics for Business Part 2

Investigation

Instructions:

1. This exam is 10% of your final grade in Mathematics.
2. This exam is 90 minutes.
3. Answer all questions.
4. Total marks = 30.
5. Show all working.
6. An approved calculator may be used.
7. No written materials or mobile phones are allowed in the examination room.

FORMULAS

Calculus – Differentiation

$f(x)$	$f'(x) = \frac{df(x)}{dx}$
x^n	nx^{n-1}
e^{kx}	$k e^{kx}$
$\ln x$	$\frac{1}{x}$

$f(x)$	$f'(x) = \frac{df(x)}{dx}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$

Chain Rule: If y is a function of u and u is a function of x , then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Product Rule: If $y = uv$, where u and v are functions of x , then

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Quotient Rule: If $y = \frac{u}{v}$ where u and v are functions of x , then

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

In questions 1-10, find $\frac{dy}{dx}$. Simplify your answers as much as possible. (2 marks each)

1. $y = \tan x - x$

2. $y = \sin x - \frac{1}{3}\sin^3 x$

3. $y = e^x \ln x$

4. $y = (\sin x) \ln x$

5. $y = \frac{1}{\cos x}$

6. $y = -\frac{1}{\sin x}$

7. $y = \frac{x}{\ln x}$

8. $y = \frac{\sin x - 1}{\cos x}$

9. $y = 2 \ln(1 + \sqrt{x})$

10. $y = \ln\left(\frac{e^x}{e^x + 1}\right)$

In questions 11-20, evaluate the integrals. Remember to include $+c$. (1 mark each)
(Hint: use your answers to questions 1-10.)

11. $\int \frac{\sin x}{\cos^2 x} dx$

12. $\int \frac{1}{x + \sqrt{x}} dx$

13. $\int \cos^3 x dx$

14. $\int \left(\frac{\sin x}{x} + (\cos x) \ln x \right) dx$

15. $\int \tan^2 x dx$

16. $\int \left(\frac{e^x}{x} + e^x \ln x \right) dx$

17. $\int \frac{\cos x}{\sin^2 x} dx$

18. $\int \frac{1}{e^x + 1} dx$

19. $\int \left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2} \right) dx$

20. $\int \frac{1}{\sin x + 1} dx$

MARK SCHEME

$$1. \frac{dy}{dx} = \sec^2 x - 1 \quad [1]$$

$$= \tan^2 x \quad [1]$$

$$2. \frac{dy}{dx} = \cos x - \sin^2 x \cos x \quad [1]$$

$$= \cos x (1 - \sin^2 x) = \cos^3 x \quad [1]$$

$$3. \frac{dy}{dx} = e^x \frac{1}{x} + e^x \ln x \quad [2]$$

$$4. \frac{dy}{dx} = (\sin x) \left(\frac{1}{x}\right) + (\cos x)(\ln x) \quad [2]$$

$$5. \frac{dy}{dx} = \frac{\sin x}{\cos^2 x} \quad [2]$$

$$6. \frac{dy}{dx} = \frac{\cos x}{\sin^2 x} \quad [2]$$

$$7. \frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2} \quad [2]$$

$$8. \frac{dy}{dx} = \frac{\cos^2 x + \sin x(\sin x - 1)}{\cos^2 x} \quad [1]$$

$$= \frac{\cos^2 x + \sin^2 x - \sin x}{\cos^2 x} = \frac{1}{1 + \sin x} \quad [1]$$

$$9. \frac{dy}{dx} = 2 \left(\frac{\frac{1}{2}x^{-\frac{1}{2}}}{1 + \sqrt{x}} \right) \quad [1]$$

$$= \frac{1}{\sqrt{x} + x} \quad [1]$$

$$10. \frac{dy}{dx} = \left(\frac{e^x + 1}{e^x} \right) \left(\frac{e^x(e^x + 1) - e^{2x}}{(e^x + 1)^2} \right) \quad [1]$$

$$= \frac{1}{e^x + 1} \quad [1]$$

$$11. \frac{1}{\cos x} + c \quad [1]$$

$$12. 2 \ln(1 + \sqrt{x}) + c \quad [1]$$

$$13. \sin x - \frac{1}{3} \sin^3 x + c \quad [1]$$

$$14. (\sin x) \ln x + c \quad [1]$$

$$15. \tan x - x + c \quad [1]$$

$$16. e^x \ln x + c \quad [1]$$

$$17. -\frac{1}{\sin x} + c \quad [1]$$

$$18. \ln \left(\frac{e^x}{e^x + 1} \right) + c \quad [1]$$

$$19. \frac{x}{\ln x} + c \quad [1]$$

$$20. \frac{\sin x - 1}{\cos x} + c \quad [1]$$

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