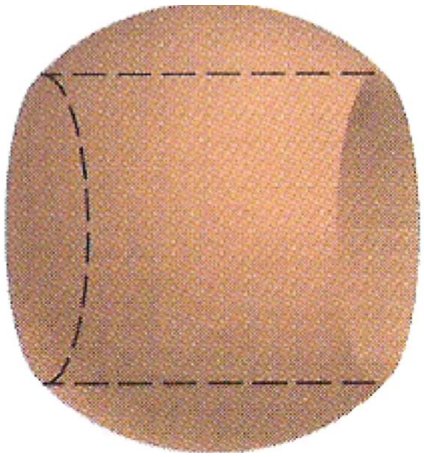
In this investigation, we will answer the following question:

If a cylindrical hole is drilled through a solid sphere so that the hole is 1 m long, what is the volume of the remaining part of the sphere?



Let $V =$ volume of the remaining part of the sphere
$A =$ volume of original sphere
$B =$ volume of cylindrical hole
$C =$ volume of each slice of the sphere that is attached to each end of the cylinder
$r =$ radius of the sphere

1. Express V in terms of A, B and C. [1]

2. Find B in terms of r. [2]

3. Superimpose *x*- and *y*-axes on the sphere so that (0,0) is at the centre of the sphere and the *x*-axis is the axis of the cylindrical hole. The outline of the cross-section of the sphere in the *x*-*y* plane is a circle.

i. Write the equation of this circle. [1]
[The equation of a circle with centre (p, q) and radius t is $(x-p)^2 + (x-p)^2 = t^2$]

ii. Use integration to find A (the volume of the original sphere) in terms of *r*. [10]
[Volume of revolution $= \pi \int_b^a y^2 \, \mathrm{d}x$]

iii. Use integration to find *C* in terms of *r*. [9]

4. Thus show that $V = \frac{1}{6}\pi$. [2]

5. Explain why *V* is independent of *r*. [2]

6. Given that *V* is independent of *r*, explain how to find *V* without finding *C*. [3]