Section A

Answer 2 Questions

A1) Solve the equation $\frac{dy}{dx} = \frac{2xy}{x^2+1}$, given that when $x = 0, y = 1$. [6]

A2) Find the general solution of $\frac{dy}{dx} - 4y = -3e^x$. [6]

A3) Find the solution of $x\frac{dy}{dx} - 2y = x^4$, given that when $x = 2$, $y = 16$. [6]

Section B

Answer 2 Questions

B1) Find the general solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$. [5]

B2) Find the general solution of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 0$. [5]

B3) Find the general solution of $25\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + 4y = 0$. [5]

Section C

Answer 2 Questions

C1) Find the general solution of $\frac{d^2v}{dx^2} + 6\frac{dv}{dx} + 34v = 26e^{-4x}$. [8]

C2) Find the general solution of $\frac{d^2w}{dx^2} + 2\frac{dw}{dx} + 5w = -5x^2 - 4x - 2$. [8]

C3) Find the general solution of $2\frac{d^2w}{dx^2} + 3\frac{dw}{dx} + w = 6\sin 2x + 7\cos 2x$. [8]

Section D

Answer 1 Question

D1) Solve the equation $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 80e^{-3t}$, given that when $t = 0, y = 8$ and $\frac{dy}{dt} = -8$. [12]

D2) Solve the equation $5\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = 15 + 12t + 5t^2$, given that when $t = 0, y = 0$ and $\frac{dy}{dt} = 0$. [12]

A1) $\frac{dy}{dx} = \frac{2xy}{x^2+1}$

$\frac{dy}{y} = \frac{2xdx}{x^2+1}$

$\int \frac{dy}{y} = \int \frac{2xdx}{x^2+1}$

$\ln y = \ln(x^2+1) + k$ Method [2] Answer [1]

$x = 0, y = 1$

$\ln 1 = \ln 1 + k$
$k = 0$

$y = x^2 + 1$ Method [2] Answer [1]

A2) $\frac{dy}{dx} - 4y = -3e^x$

characteristic equation

$m - 4 = 0; m = 4$

$y_c = Ae^{4x}$ Method [2] Answer [1]

$y_p = Ce^x; \quad -4y_p = -4Ce^x$

$y_p' = Ce^x; \quad y_p' = Ce^x$

$y_p' - 4y_p = -3Ce^x = -3e^x; \qquad C = 1$

$y_p = e^x$ Method [2] Answer [1]

$y = y_c + y_p = Ae^{4x} + e^x$.

OR by integrating factor.

A3) $x\frac{dy}{dx} - 2y = x^4$

$\frac{dy}{dx} - 2\frac{y}{x} = x^3$

$e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = 1/x^2$ Method [1] Answer [1]

$\frac{1}{x^2}\left\{\frac{dy}{dx} - 2\frac{y}{x}\right\} = x$

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = x$$

$$\frac{y}{x^2} = \int x dx = \frac{x^2}{2} + c$$ Method [1] Answer [1]

$$y = \frac{x^4}{2} + cx^2$$

$$x = 2, y = 16$$

$$16 = 8 + c4; c = 2$$

$$y = \frac{x^4}{2} + 2x^2.$$ Method [1] Answer [1]

B1) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

$$m^2 + 3m + 2 = 0$$

$$(m+2)(m+1) = 0$$

$$m = -1, m = -2$$

$$y = Ae^{-x} + Be^{-2x}.$$ Method [3] Answer [2]

B2) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 0$

$$m^2 + 4m + 29 = 0$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4.29}}{2} = -2 \pm 5i$$

$$y = e^{-2x}(A\cos 5x + B\sin 5x).$$ Method [3] Answer [2]

B3) $25\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + 4y = 0$

$$25m^2 - 20m + 4 = 0$$

$$m = \frac{20 \pm \sqrt{20^2 - 4.25.4}}{50} = 2/5$$

$y = (A + Bx)e^{-2x/5}$. Method [3] Answer [2]

C1) $\frac{d^2v}{dx^2} + 6\frac{dv}{dx} + 34v = 26e^{-4x}$

$m^2 + 6m + 34 = 0$

$m = \frac{-6 \pm \sqrt{6^2 - 4.34}}{2} = -3 \pm 5i$

$v_c = e^{-3x}(A\cos 5x + B\sin 5x)$ Method [2] Answer [2]

$v_p = Ce^{-4x}$

$34v_p = 34Ce^{-4x}$

$v_p^{'} = -4Ce^{-4x}$

$6v_p^{'} = -24Ce^{-4x}$

$v_p^{''} = 16Ce^{-4x}$

$v_p^{''} + 6v_p^{'} + 34v_p = 16Ce^{-4x} - 24Ce^{-4x} + 34Ce^{-4x} = 26Ce^{-4x} = 26e^{-4x}$

$C = 1$

$v_p = e^{-4x}$ Method [2] Answer [2]

$v = v_c + v_p = e^{-3x}(A\cos 5x + B\sin 5x) + e^{-4x}$.

C2) $\frac{d^2w}{dx^2} + 2\frac{dw}{dx} + 5w = -5x^2 - 4x - 2$

$m^2 + 2m + 5 = 0$

$m = \frac{-2 \pm \sqrt{2^2 - 4.5}}{2} = -1 \pm 2i$

$w_c = e^{-x}(A\cos 2x + B\sin 2x)$ Method [2] Answer [2]

$w_p = Cx^2 + Dx + E$

$w_p^{'} = 2Cx + D$

$w_p^{''} = 2C$

$w_p'' + 2w_p' + 5w_p = 5Cx^2 + 5Dx + 5E + 4Cx + 2D + 2C = -5x^2 - 4x - 2$

$x^2 : Cx^2 = -5x^2; C = -1$

$x : 5Dx - 4 = -4; D = 0$

$constant : 5E + 2.0 - 2 = -2; E = 0$

$w = w_c + w_p = e^{-x}(A\cos 2x + B\sin 2x) - x^2$. Method [2] Answer [2]

C3) $2\frac{d^2w}{dx^2} + 3\frac{dw}{dx} + w = 6\sin 2x + 7\cos 2x$

$2m^2 + 3m + 1 = 0$

$(2m+1)(m+1) = 0$

$m = -1, m = -1/2$

$w_c = Ae^{-x} + Be^{-x/2}$ Method [2] Answer [2]

$w_p = C\sin 2x + D\cos 2x$

$w_p' = -2D\sin 2x + 2C\cos 2x$

$w_p'' = -4C\sin 2x - 4D\cos 2x$

$w_p + 3w_p' + 2w_p'' = (-7C - 6D)\sin 2x + (6C - 7D)\cos 2x = 6\sin 2x + 7\cos 2x$

$-7C - 6D = 6$
$6C - 7D = 7$

$C = 0, D = -1$

$w_p = -\cos 2x$ Method [2] Answer [2]

$w = w_c + w_p = Ae^{-x} + Be^{-x/2} - \cos 2x$

D1) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 80e^{-3t}$

$m^2 + 6m + 25 = 0$

$m = \frac{-6 \pm \sqrt{6^2 - 4.25}}{2} = -3 \pm 4i$

$y_c = e^{-3t}(A\cos 4t + B\sin 4t)$ Method [2] Answer [2]

$y_p = Ce^{-3t}$

$y_p' = -3Ce^{-3t}$

$y_p'' = 9Ce^{-3t}$

$y_p'' + 6y_p' + 25y_p = Ce^{-3t} - 18Ce^{-3t} + 25Ce^{-3t} = 16Ce^{-3t} = 80Ce^{-3t}$

$C = 5$

$y = y_c + y_p = e^{-3t}(A\cos 4t + B\sin 4t + 5)$ Method [2] Answer [2]

$y(0) = A + 5 = 8$

$A = 3$

$y' = -3e^{-3t}(A\cos 4t + B\sin 4t + 5) + e^{-3t}(-4A\sin 4t + 4B\cos 4t)$

$y'(0) = -3(A+5) + 4B = -8$

$B = 4$

$y = e^{-3t}(3\cos 4t + 4\sin 4t + 5)$. Method [2] Answer [2]

D2) $5\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 5y = 15 + 12t + 5t^2$

$5m^2 + 6m + 5 = 0$

$m = \dfrac{-6 \pm \sqrt{6^2 - 4.5.5}}{10} = -\dfrac{3}{5} \pm \dfrac{4}{5}i$

$y_c = e^{-3t/5}(A\cos\frac{4}{5}t + B\sin\frac{4}{5}t)$ Method [2] Answer [2]

$y_p = Ct^2 + Dt + E$

$y_p' = 2Ct + D$

$y_p'' = 2C$

$5y_p'' + 6y_p' + 5y_p = 5Ct^2 + 5Dt + 5E + 12Ct + 6D + 10C = 5t^2 + 12t + 15$

$x^2 : 5Ct^2 = 5t^2$

$C = 1$

$x: 5Dt + 12Ct = 12t$

$D = 0$

$const: 5E + 6D + 10C = 15$

$E = 1$

$y_p = t^2 + 1$ Method [2] Answer [2]

$y = y_c + y_p = e^{-3t/5}(A\cos\frac{4}{5}t + B\sin\frac{4}{5}t) + t^2 + 1$

$y(0) = A + 1 = 0$

$A = -1$

$y' = -\frac{3}{5}e^{-3t/5}(A\cos\frac{4}{5}t + B\sin\frac{4}{5}t) + e^{-3t/5}(-\frac{4}{5}A\sin\frac{4}{5}t + \frac{4}{5}B\cos\frac{4}{5}t) + 2t$

$y'(0) = -\frac{3}{5}A + \frac{4}{5}B = 0$

$B = -\frac{3}{4}$

$y = e^{-3t/5}(-\cos\frac{4}{5}t - \frac{3}{4}\sin\frac{4}{5}t) + t^2 + 1.$ Method [2] Answer [2]