NCUK
THE NORTHERN CONSORTIUM

Semester 2 Examination

Further Mathematics : Vectors

Examination Session
May 2012

Time Allowed
1 hour

INSTRUCTIONS TO STUDENTS

- **Write your Student Number clearly on the Answer Booklet Provided**

1 **This exam is worth 5% of the overall marks for the course.**
2 **The time allowed for this exam is 1 hour.**
3 **This paper contains 9 questions.**
4 **Answer all questions.**
5 **The total number of marks for the exam is 50.**
6 **The marks for each question are indicated in square brackets.**
7 **Only approved calculators may be used.**
8 **No written material is allowed in the examination room.**
9 **No mobile phones are allowed in the examination room**

Vectors

Unit vector $\hat{\mathbf{a}}$ in the direction of $\mathbf{a}$

$\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$, where $|\mathbf{a}|$ is the modulus (magnitude) of $\mathbf{a}$.

$\vec{a}$ and $\overrightarrow{AB}$ are also used to denote vectors.

Scalar Product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{a}$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$,
then $\mathbf{i}.\mathbf{i} = \mathbf{j}.\mathbf{j} = \mathbf{k}.\mathbf{k} = 1$, $\mathbf{i}.\mathbf{j} = \mathbf{j}.\mathbf{k} = \mathbf{k}.\mathbf{i} = 0$,
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$,
$\mathbf{a}.\mathbf{a} = |\mathbf{a}|^2 = {a_1}^2 + {a_2}^2 + {a_3}^2$.
If both $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors then $\mathbf{a}$ is perpendicular to $\mathbf{b}$ if $\mathbf{a}.\mathbf{b} = 0$.

Vectors

Vector Product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{n}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$,
then $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$, $\mathbf{i} \times \mathbf{j} = \mathbf{k}$, $\mathbf{j} \times \mathbf{k} = \mathbf{i}$, $\mathbf{k} \times \mathbf{i} = \mathbf{j}$,
$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$.

If both $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors then $\mathbf{a}$ is parallel to $\mathbf{b}$ if $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

Moments as vectors

The moment about O of force $\mathbf{F}$ acting at position $\mathbf{r}$ is $\mathbf{r} \times \mathbf{F}$.

A point is given by the position vector $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

A line is given by the vector equation l: $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$.

A plane is given by the vector equation Π: $\mathbf{r}.(4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6$.

1) Find the Cartesian equation of the plane. [3]

2) Find the Cartesian equation of the line. [3]

3) Find the distance of the point from the plane, giving your answer in exact form. [5]

4) Find the point of intersection of the line and the plane. [6]

5) Find the angle of intersection of the line and the plane, giving your answer in degrees to 3 significant figures. [3]

6) Find the equation of the plane containing the point and the line, giving your answer in the form $\mathbf{r.n} = D$. [8]

7) Find the distance of the point from the line, giving your answer in exact form. [7]

8) Consider any 4 distinct points, A, B, C and D.

Let AB = **a**, BC = **b**, CD = **c** and DA = **d**.

i) Find $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d}$.

Let W be the midpoint of AB, X be the midpoint of BC, Y be the midpoint of CD and Z be the midpoint of DA.

ii) Find WX and ZY in terms of **a** and **b**.

iii) Find XY and WZ in terms of **b** and **c**.

iv) What can you say about WXYZ? [6]

9) Consider the points A = 2**i** + **j** +3**k**, B = **i** - 2**j** + **k**, C = 4**i** - 3**j** - **k** and D = -2**i** + 3**j** - 2**k**.

i) Find the area of the triangle ABC

ii) Find the volume of the tetrahedron ABCD. [9]