NCUK
THE NORTHERN CONSORTIUM

Semester 2 Examination

Further Mathematics : Vectors

Examination Session
May 2011

Time Allowed
1 hour

INSTRUCTIONS TO STUDENTS

- **Write your Student Number clearly on the Answer Booklet Provided**

1 This exam is worth 5% of the overall marks for the course.
2 The time allowed for this exam is 1 hour.
3 This paper contains 6 questions.
4 Answer 4 questions.
5 The total number of marks for the exam is 100.
6 The marks for each question are indicated in square brackets.
7 Only approved calculators may be used.
8 No written material is allowed in the examination room.
9 No mobile phones are allowed in the examination room

Vectors

Unit vector $\hat{\mathbf{a}}$ in the direction of $\mathbf{a}$

$\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$, where $|\mathbf{a}|$ is the modulus (magnitude) of $\mathbf{a}$.

$\vec{a}$ and $\overrightarrow{AB}$ are also used to denote vectors.

Scalar Product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{a}$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$,
then $\mathbf{i}.\mathbf{i} = \mathbf{j}.\mathbf{j} = \mathbf{k}.\mathbf{k} = 1$, $\mathbf{i}.\mathbf{j} = \mathbf{j}.\mathbf{k} = \mathbf{k}.\mathbf{i} = 0$,
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$,
$\mathbf{a}.\mathbf{a} = |\mathbf{a}|^2 = {a_1}^2 + {a_2}^2 + {a_3}^2$.
If both $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors then $\mathbf{a}$ is perpendicular to $\mathbf{b}$ if $\mathbf{a}.\mathbf{b} = 0$.

Vectors

Vector Product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{n}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$,
then $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$, $\mathbf{i} \times \mathbf{j} = \mathbf{k}$, $\mathbf{j} \times \mathbf{k} = \mathbf{i}$, $\mathbf{k} \times \mathbf{i} = \mathbf{j}$,
$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$.

If both $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors then $\mathbf{a}$ is parallel to $\mathbf{b}$ if $\mathbf{a} \times \mathbf{b} = \mathbf{0}$.

Moments as vectors

The moment about O of force $\mathbf{F}$ acting at position $\mathbf{r}$ is $\mathbf{r} \times \mathbf{F}$.

1) Consider the points with position vectors $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{c} = \mathbf{i} - \mathbf{j} - \mathbf{k}$.
a) Find the area of the triangle defined by the points. [10]
b) Find the equation of the plane defined by the points in the form $\mathbf{r}.\mathbf{n} = D$. [15]
Show full workings

2) Lines $\mathbf{r} = (2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ lie in the same plane.
a) Find the equation of the plane in the form $\mathbf{r.n} = D$ [15]
b) Find the distance of the plane from the origin. [10]
Show full workings.

3) A line is given by $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$ and a plane is given by $\mathbf{r}.(\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 6$.
a) Find the angle between the line and the plane, in degrees, correct to 3 significant figures. [10]
b) Find the point of intersection of the line and the plane. [15]
Show full workings.

4) A plane is given by $\mathbf{r}.(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) = 7$ and a point has a position vector $\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.
a) Find the distance of the point from the plane. [15]
b) Find the cartesian form of the equation of the plane. [10]
Show full workings.

5) Two planes are given by $\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) = 3$ and $\mathbf{r}.(\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 6$.
a) Find the angle of intersection of the planes giving your answer to the nearest degree. [10]
b) Find the equation of the line of intersection of the planes. [15]
Show full workings.

6)
a) Find the volume of the parallelpiped with a vertex at the origin and three of whose edges are given by OA = $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, OB = $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and OC = $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$. [10]
b) Find the volume of the tetrahedron whose four vertices are given by A = $(-2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$, B = $(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$, C = $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k})$ and D = $(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$.
Show full workings. [15]