NCUK
THE NORTHERN CONSORTIUM

Semester 2 Examination

Further Mathematics : Differential Equations

Examination Session
May 2011

Time Allowed
1 hour

INSTRUCTIONS TO STUDENTS

- **Write your Student Number clearly on the Answer Booklet Provided**

1 **This exam is worth 5% of the overall marks for the course.**
2 **The time allowed for this exam is 1 hour.**
3 **This paper contains 4 sections.**
4 **Answer 2 questions from section A, 2 questions from section B, 2 questions from section C and 1 question from section D.**
5 **Give all answers in the from y = f(x).**
6 **Show full workings.**
6 **The total number of marks for the exam is 50.**
7 **The marks for each question are indicated in square brackets.**
8 **Only approved calculators may be used.**
9 **No written material is allowed in the examination room.**
10 **No mobile phones are allowed in the examination room.**

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Section A. Answer 2 questions.

1) Solve $\frac{dy}{dx}+2\frac{y}{x}=6x^3$. [6]

2) Solve $\frac{dy}{dx}+3y=4x$. [6]

3) Solve $\frac{dy}{dx}=-k(y+5)$, given that when $x=0$, $y=3$ and $\frac{dy}{dx}=-16$. [6]

Section B. Answer 2 questions.

1) Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}-12y=0$. [5]

2) Solve $9\frac{d^2y}{dx^2}-12\frac{dy}{dx}+4y=0$. [5]

3) Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+13y=0$. [5]

4) Show that $y=A\sinh 2x+B\cosh 2x$ is a solution of $\frac{d^2y}{dx^2}-4y=0$. [5]

Section C. Answer 2 questions.

1) Solve $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=10e^{3x}$. [8]

2) Solve $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=30x^2-2x+3$. [8]

3) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=15\sin 2x-10\cos 2x$. [8]

Section D. Answer 1 question.

1) Solve $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=-6x^2+22x$, given that when $x=0, y=3$ and $\frac{dy}{dx}=12$.

[12]

2) Solve $\frac{d^2y}{dx^2}-7\frac{dy}{dx}+12y=-2e^{3x}$, given that when $x=0, y=-1$ and $\frac{dy}{dx}=1$.

[12]

D.E. Exam Answers -- May 2011

A1) $y'+2y/x=6x^3$

$y = x^4 + c/x^2$.

A2) $y'+3y = 4x$

Using the characteristic equation or an integrating factor gives,

$y = yc + yp = Ae^{-3x} + 4x/3 - 4/9$.

A3) $y' = -k(y+5)$

$y = 8e^{-2x} - 5$.

B1) $y''-y'-12y = 0$

$y = Ae^{4x} + Be^{-3x}$

B2) $9y''-12y'+4y = 0$

$y = (A+Bx)e^{2x/3}$

B3) $y''-6y'+13y = 0$

$y = e^{3x}(A\cos 2x + B\sin 2x)$

B4)

$y = A\sinh 2x + B\cosh 2x$

$y' = 2A\cosh 2x + 2B\sinh 2x$

$y'' = 4A\sinh 2x + 4B\cosh 2x$

$y''-4y = 0,$ as required.

C1) $y''-6y'+9y = 10e^{3x}$

$y = y_c + y_p = (A+Bx+5x^2)e^{3x}$

C2) $y''-8y'+15y = 30x^2 - 2x + 3$

$y = Ae^{3x} + Be^{5x} + 2x^2 + 2x + 1.$

C3) $y''-2y'-3y = 15\sin 2x - 10\cos 2x$

$y = Ae^{3x} + Be^{-x} - \sin 2x + 2\cos 2x$.

D1) $y''-y'-6y = -6x^2 + 22x$

$y = 4e^{3x} - 2e^{-2x} + x^2 - 4x + 1$.

D2) $y''-7y'+12y = -2e^{3x}$

$y = (7 + 2x)e^{3x} - 8e^{4x}$